Economathematics

Problem Sheet 5

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1. Consider the Black-Scholes model and the derivative asset:

$$X = \begin{cases} 0 & S_T \leqslant A, \\ S_T - A & A < S_T \leqslant K + A, \\ A + 2K - S_T & K + A < S_T \leqslant \frac{3}{2}K + A, \\ \frac{1}{2}K & S_T > A + \frac{3}{2}K. \end{cases}$$

Replicate this derivative using portfolio consisting of bond, asset S and European option. Find the arbitrage free price for X.

2. Consider the Black-Scholes model and the derivative asset:

$$X = \begin{cases} 0 & S_T \leqslant A, \\ S_T - A & A < S_T \leqslant K + A, \\ K & A + K < S_T \leqslant B + K + A, \\ A + K + B - S_T & K + A + B < S_T \leqslant 2K + A + B \\ 0 & S_T > A + 2K + B \end{cases}$$

Replicate this derivative using portfolio consisting of bond, asset S and European option. Find the arbitrage free price for X.

3. Find the arbitrage free price of $X = (S_T)^{T/2}$ for Black-Scholes market with expiry date T.

4. Find the arbitrage free price of $X = S_T/S_{T_0}$ for Black-Scholes market with expiry date T.

5. Find the arbitrage free price of $X = S_T \mathbf{1}_{\{S_T \in [a,b]\}}$ for Black-Scholes market with expiry date T.

6. Find the arbitrage free price of $X = S_T - \sqrt{S_T}$ for Black-Scholes market with expiry date T.

7. A non-dividend-paying stock, currently priced at \$125 per share, can either go up by \$25 or down \$25 in a year. Consider a one-year European call option with a strike price of \$135. The continuously-compounded risk-free interest rate is 8%. Use a one-period binomial model to determine the current price of the call option.

8. Consider a non-dividend-paying stock with the initial price of \$100. Assume that the annual risk-free continuously compounded interest rate equals $r = 0.05$. Let the annualized standard deviation of the continuously compounded stock return, i.e., the volatility be $\sigma = 0.25$. Using a one-period forward binomial tree, calculate the price of a one-year at-the-money European call on this underlying asset.

9. (Problem for 20 points) Consider a two-period binomial model for the stock price with both periods of length one year. Let the initial stock price be 100 and assume that the stock pays no dividends. Let the up and down factors be $u = 1.25$ and $d = 0.75$, respectively. Let the continuously compounded interest rate be $r = 0.05$ per annum. Roger is interested in purchasing a chooser option with the provision that he can choose if the option is a put or a call after one year. The strike for this option is 100 and the expiry date is two years. Using the above binomial tree, find the price of the chooser option.

10. (Problem for 20 points) For a two-period binomial model, you are given that:

- each period is one year;
- the current price of a non-dividend-paying stock S is 20;
- $u = 1.3$, with u as in the standard notation for the binomial model;
- $d = 0.9$, with d as in the standard notation for the binomial model;
- the continuously compounded risk-free interest rate is $r = 0.05$.

Find the price of an American call option on the stock with $T = 2$ and the strike price $K = 23$