EXERCISE:
Construct a subshift (X', T) with the following three properties

1) X' contains a fixpoint which is the unique minimal set;

2) X' has an invariant measure of full topological support;

3) (X', T) is topologically mixing.

(T will denote the shift map, regardless of the alphabet, subshift, etc.)

We begin with any strictly ergodic subshift (X, T, μ). Choose clopen sets $U_k \searrow \{x^*\}$ such that:

1) $\sum_{k\geq 1} \mu(U_k) < \infty$

2) $U_k, T(U_k), \ldots, T^{2lk}(U_k)$ are pairwise disjoint, where l is the syndetic constant of U_1.

Then we create x' from $x \in X$:

$$x = \ldots x_{n_{-1}} x_{n_{-1}+1} \ldots x_0 x_1 \ldots x_{n_0} x_{n_0+1} \ldots x_{n_1} x_{n_1+1} \ldots$$

$$x' = \ldots x_{n_{-1}} \underbrace{ccc\ldots c}_{k_{-1}\ or\ k_{-1}+1} x_{n_{-1}+1} \ldots x_0 x_1 \ldots x_{n_0} \underbrace{ccc\ldots c}_{k_0\ or\ k_0+1} x_{n_0+1} \ldots x_{n_1} \underbrace{ccc\ldots c}_{k_1\ or\ k_1+1} x_{n_1+1} \ldots$$

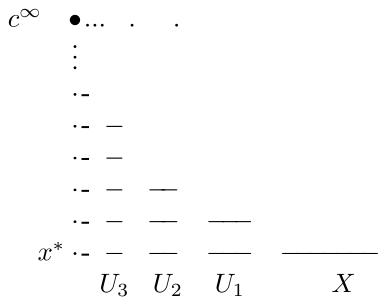
$$x^* = \ldots x_{n_{-1}} x_{n_{-1}+1} \ldots x_{n_0=0} x_1 \ldots x_{n_1} x_{n_1+1} \ldots \qquad (T^{-n}(x^*), n > 0 \text{ alike})$$

$$x' = \ldots x_{n_{-1}} \underbrace{ccc\ldots c}_{k_{-1}\ or\ k_{-1}+1} x_{n_{-1}+1} \ldots x_{n_0=0} \underbrace{cccccccccccc\ldots}_{k_0=\infty}$$

$$T(x^*) = \ldots x_{n_{-1}} x_{n_{-1}+1} \ldots x_{n_0} x_{n_0+1=0} \ldots x_{n_1} x_{n_1+1} \ldots \qquad (T^n(x^*), n > 1 \text{ alike})$$

$$x' = \underbrace{\ldots cccccccccccc}_{n_0=\infty} x_{n_0+1=0} \ldots x_{n_1} \underbrace{ccc\ldots c}_{k_1\ or\ k_1+1} x_{n_1+1} \ldots$$

The topological structure of X' (simplification)

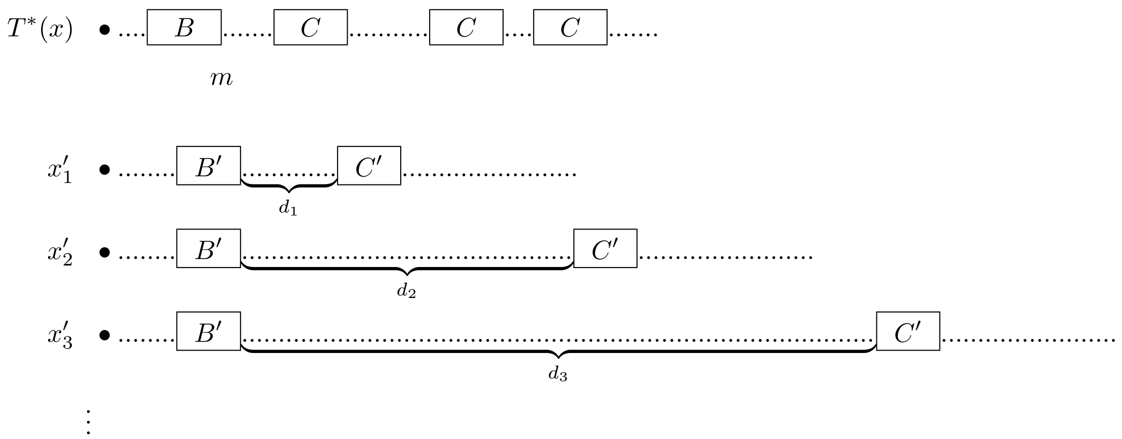


Up to a countable set of nonisolated points, the true base has the structure of a skew product:

$$
\begin{aligned}
&(\, X \times \{0,1\}^{\mathbb{Z}},\ S\,) \\
&S(x,a) = (T(x), T_x(a))
\end{aligned}
$$

$$
T_x = \begin{cases} T; & x \in U_1 \\ \mathrm{Id}; & x \notin U_1 \end{cases}
$$

$\mu \times \lambda$ is invariant (ergodic) and has full support.



$T^*(x)$
B
C
C
C
m
x'_1
B'
C'
d_1
x'_2
B'
C'
d_2
x'_3
B'
C'
d_3