# Unit 5
# Absolute Value

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## 1 Absolute value and its properties

*An absolute value* $|x|$ of a real number $x$ is its distance from the point 0 on a number line. More specifically,

$$|x| = \begin{cases} x & \text{for } x \geqslant 0, \\ -x & \text{for } x < 0. \end{cases} \tag{1}$$

Most important properties of the absolute value are listed below

$$|x| \geqslant 0 \qquad \text{for all } x \in \mathbb{R}, \tag{2}$$
$$|-x| = |x| \qquad \text{for all } x \in \mathbb{R}, \tag{3}$$
$$\sqrt{x^2} = |x| \qquad \text{for all } x \in \mathbb{R}, \tag{4}$$
$$|x \cdot y| = |x| \cdot |y| \qquad \text{for all } x, y \in \mathbb{R}, \tag{5}$$
$$\left|\frac{x}{y}\right| = \frac{|x|}{|y|} \qquad \text{for all } x, y \in \mathbb{R}, y \neq 0, \tag{6}$$
$$|x + y| \leqslant |x| + |y| \qquad \text{for all } x, y \in \mathbb{R}. \tag{7}$$

### Exercises

**1.1.** Compute

a) $|-5|$,

b) $|\sqrt{3}|$,

c) $|2 + \sqrt{2}|$,

d) $|2 - \sqrt{2}|$,

e) $|1+2\sqrt{2}|$,

f) $|1-2\sqrt{2}|$,

g) $\sqrt{(-8)^2}$,

h) $\sqrt{(1-\sqrt{3})^2}$,

i) $|(2+\sqrt{3})(1-\sqrt{2})|$,

j) $\left|\frac{1-\sqrt{5}}{2}\right|$.

**1.2.** Draw the graphs of the following functions

a) $f(x)=|x|$,

b) $f(x)=|2x|$,

c) $f(x)=2|x|$,

d) $f(x)=\left|\frac{1}{4}x\right|$,

e) $f(x)=3|x-2|$,

f) $f(x)=2|x+1|+4$,

g) $f(x)=|x-3|+|x+2|$,

h) $f(x)=\frac{|x|}{x}$.

**1.3.** Solve the following equations

a) $|x|=3$,

b) $|x|=-3$,

c) $|x-1|=2$,

d) $|4-x|=1$,

e) $|3x-6|=6$,

f) $|7-2x|=1$,

g) $|x|=2x$,

h) $2|x-3|=1-x$,

i) $\Big||x|-2\Big|=3$,

j) $\Big||x-1|+4\Big|=2$,

k) $\Big|2-|5-x|\Big|=1$,

l) $\Big||3-2x|+1\Big|=4$,

m) $|x-1|+|x+2|=3$,

n) $|x+1|=2-|x-1|$,

o) $6-|4-x|=|2-3x|$,

**1.4.** Solve the following inequalities

a) $|x|>3$,

b) $|x|\leqslant 2$,

c) $|x-1|\geqslant 3$,

d) $|6-x|<4$,

e) $|2x-4|>8$,

f) $|9-3x|\leqslant 1$,

g) $|x+3|\geqslant 2x-6$

h) $2|x-3|<1-x$,

i) $|x|+2x>2$,

j) $\Big||x|+4\Big|<12$,

k) $\Big|2|x-1|-3\Big|\leqslant 5$,

l) $\Big|3-|x-2|\Big|\geqslant 2$,

m) $\Big||4-2x|-2\Big|\geqslant 8$,

n) $|x-3|+|x+5|\leqslant 10$,