Exercise Wroclaw 22.10.2018

I. **Bernoulli Trials: Even odds.**

i. Presume you are playing Blackjack against a dealer in a casino. Even odds imply that given that you bet a certain amount of money, you lose either everything or double your stake, which starts with 100. Without card counting, your winning probability is p=45% per trial.

a) Plot the expected logarithm of wealth g(f), as a function of the investment fraction (% of wealth). Which investment fraction would you bet?

b) Simulate n=1000 trials consecutively. How does your wealth evolve over the number of trials? Compare it to the strategy maximizing your expectation.

c) Simulate n=1000 trials independently. Draw a boxplot of the 1000 final wealths. Compare it to the strategy maximizing your expectation.

ii. As you learned to start counting cards, your winning probability increased to p=52%. Redo a), b) and c) from i.

II. **Bernoulli Trials: Uneven odds.**

i. A friend of yours gave you a tip for a horse race. The odds are 2:1 implying that you either win two times your stake, or lose everything. Derive the optimal betting fraction f. Refer to Thorp (2007) if necessary.

ii. (optional) Modify your code from I as such the odds are one further input parameter and plot a), b) and c).

III. Gaussian trials

i. Assume you wanted to invest in the stock market. Under Gaussianity, you estimate the mean of the log returns to be $\mu = 6\%$ p.a., the standard deviation to be $\sigma = 20\%$ p.a. and the risk free rate is r=1% p.a.

   a) Calculate the growth-optimal investment fraction utilizing the closed-form solution from Merton.

   b) Plot the expected logarithm of wealth g(f), as a function of the investment fraction (% of wealth). Do not use the approximation, simulate returns instead (Note the difference between discrete and log returns).

   c) Simulate n=1000 years consecutively. How does your wealth evolve over the number of trials?

   d) Simulate n=1000 years independently. Draw a boxplot of the 1000 final wealths. Estimate arithmetic return, geometric return, Value at Risk for confidence levels 95% and 99% for the discrete wealth returns.

ii. Change the underlying distribution assumption to Student-t ($\nu = 5$). How does the optimal fraction change as ν increases / decreases.