Day-ahead electricity price forecasting with high-dimensional structures: Univariate vs. multivariate modeling frameworks*

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Abstract

We conduct an extensive empirical study on short-term electricity price forecasting (EPF) to address the long-standing question if the optimal model structure for EPF is univariate or multivariate. We provide evidence that despite a minor edge in predictive performance overall, the multivariate modeling framework does not uniformly outperform the univariate one across all 12 considered datasets, seasons of the year or hours of the day, and at times is outperformed by the latter. This is an indication that combining advanced structures or the corresponding forecasts from both modeling approaches can bring a further improvement in forecasting accuracy. We show that this indeed can be the case, even for a simple averaging scheme involving only two models. Finally, we also analyze variable selection for the best performing high-dimensional lasso-type models, thus provide guidelines to structuring better performing forecasting model designs.

Keywords: Electricity price forecasting, Day-ahead market, Univariate modeling, Multivariate modeling, Forecast combination, Regression, Variable selection, Lasso

1. Introduction

There is no consensus in the existing literature on short-term electricity price forecasting (EPF) as to the representation of the price series (see Weron, 2014, for a recent review). Should the modeling be implemented in a multivariate fashion, i.e., with separate but possibly interdependent models for each of the 24 (48 or more) load periods, or within a univariate framework, where one large model is constructed and the same set of parameters is used to produce one- to 24-step ahead predictions for all load periods of the next day?

Surprisingly, though, there are very few and very limited studies in the EPF literature where the univariate and multivariate frameworks are compared. Cuaresma et al. (2004) apply variants of AR(1) and general ARMA processes (including ARMA with jumps) to short-term EPF in the German EEX market. They conclude that specifications in which each hour of the day is modeled separately (i.e., a multivariate framework) present uniformly better forecasting properties than univariate time series models. More recently, Ziel (2016a) notes that, when we compare the forecasting performance of relatively simple time series models implemented either in a multivariate
or a univariate framework, the latter generally perform better for the first half of the day, whereas
the former are better in the second half of the day. However, there has been no thorough, empirical
study to date, involving many fine-tuned specifications from both groups. With this paper we want
to fill the gap and provide much needed evidence. In particular we want to address three pertinent
questions:

1. Which modeling framework – multivariate or univariate – is better for EPF?
2. If one of them is better, is it better across all hours, seasons of the year and markets?
3. How many and which past values of the spot price process should be used in EPF models?

The remainder of the paper is structured as follows. In Section 2 we thoroughly discuss the
univariate and multivariate modeling frameworks, which are driven by different data-format per-
spectives. This is a crucial, conceptual part of the paper, which sets ground for the empirical
analysis in the following Sections. In Section 3 we briefly describe the 12 price series used and
present the area hyperbolic sine transform for stabilizing the variance of spot price data. In Section
4 we define 10 forecasting models representing eight model classes: (C1) the mean values of the
past prices, (C2) similar-day techniques, (C3) sets of 24 parsimonious, interrelated autoregres-
sive (AR) structures (so-called expert models), (C4) sets of 24 univariate AR models, (C5) vector
autoregressive (VAR) models, (C6) sets of 24 parameter-rich, interrelated AR models estimated
using the least absolute shrinkage and selection operator (i.e., lasso or LASSO; which shrinks to zero the coefficients of redundant explanatory variables), (C7) univariate AR models and (C8)
univariate, parameter-rich AR models estimated using the lasso. In Section 5 we evaluate their
performance on the basis of the Mean Absolute Error (MAE), the mean percentage deviation from
the best (m.p.d.f.b.) model and using two variants of the Diebold and Mariano (1995) test for
significant differences in the forecasting performance. We also discuss variable selection for the
best performing lasso-type models. In Section 6 we wrap up the results and provide guidelines for
energy modelers and forecasters. Finally, in the Appendixes we define the full set of 58 forecasting
models considered in our empirical study (for clarity of exposition in Section 5 we report detailed
results only for 10 representative models), provide formulas for alternative representations of some
of the models, and summarize the predictive performance of all 58 models.

2. The univariate and multivariate modeling frameworks

Recall, that the day-ahead price series is a result of conducted once per day (usually around
noon) auctions for the 24 hours of the next day (Burger et al., 2007; Huisman et al., 2007; Veron
and Ziel, 2018). Consequently, the electricity prices \( P_{d,1}, \ldots, P_{d,24} \) for day \( d \) and hours \( 1, \ldots, 24 \)
are disclosed at once, and can be regarded as a multivariate time series of the 24-dimensional
random vector \( P_d = [P_{d,1}, \ldots, P_{d,24}] \). Next to the daily auction argument there are two other
practical reasons for the multivariate modeling framework: (i) the demand forecasting literature,
which has generally favored the multivariate framework for short-term predictions, and (ii) the fact
that each load period (hour, half-hour) displays a rather distinct price profile, reflecting the daily
variation of demand, costs, operational constraints and bidding strategies (Gianfreda et al., 2016;
Karakatsani and Bunn, 2008; Shahidehpour et al., 2002). On the other hand, the electricity prices
can be rewritten as one ‘high-frequency’ (hourly, half-hourly) univariate time series: \( P_t = P_{24d+h} = \)
P_{d,h}\), hence are prone to modeling within a univariate framework. The univariate approach is more popular in the engineering EPF literature, dominated by neural network models (see Aggarwal et al. 2009 for a review), but has its roots also in the traditional time series analysis of financial and commodity markets.

Both approaches have their proponents. For instance, Cuaresma et al. (2004), Misiorek et al. (2006), Zhou et al. (2006), Garcia-Martos et al. (2007), Karakatsani and Bunn (2008), Lisi and Nan (2014), Alonso et al. (2016), Gaillard et al. (2016), Hagfors et al. (2016), Maciejowska et al. (2016), Nowotarski and Weron (2016), Uniejewski et al. (2016), among others, advocate the use of sets of 24 (48 or more) models estimated independently for each load period, typically using Ordinary Least Squares (OLS). In the neural network literature, Amjady and Keynia (2009a), Marcjasz et al. (2018) and Panapakidis and Dagoumas (2016), among others, use a separate network (i.e., a different parameter set) for each hour of the next day.

Studies where univariate statistical time series models are used include Nogales et al. (2002), Contreras et al. (2003), Conejo et al. (2005), Zareipour et al. (2006), Paraschiv et al. (2015) and Ziel et al. (2015a), while papers where neural networks are put to work include Rodriguez and Anders (2004), Amjady (2006), Pao (2007), Amjady et al. (2010), Abedinia et al. (2015), Kim (2015), Dudek (2016), Keles et al. (2016) and Rafiei et al. (2017), among others.

2.1. The multivariate modeling framework

The simplest, yet surprisingly often used structure for the 24-dimensional price time series is a set of 24 univariate models:

\[
\begin{align*}
P_{d,1} &= f_1(P_{d-1,1}, P_{d-2,1}, \ldots) + \varepsilon_{d,1} \rightarrow \hat{P}_{d,1}, \\
&\vdots \\
P_{d,24} &= f_{24}(P_{d-1,24}, P_{d-2,24}, \ldots) + \varepsilon_{d,24} \rightarrow \hat{P}_{d,24},
\end{align*}
\]

where \(\varepsilon_{d,h}\) is the innovation (noise) term for day \(d\) and hour \(h\), and \(f_h(\cdot)\) are some functions of the explanatory variables of the past prices in the same load period. A commonly raised argument in favor of this approach is that it is simple to implement, involves only a small number of parameters for each load period and hence is computationally non-demanding. The downside, however, is that the estimated set of models does not take into account the potentially important dependencies between the variables across the load periods. Still, by increasing the set of dependent explanatory variables such interrelationships can be added. For instance, Gaillard et al. (2016), Uniejewski et al. (2016) and Ziel (2016a) consider the previous day’s price for midnight, i.e., \(P_{d-1,24}\), as an explanatory variable in each of the 24 single models. Formally such a set of 24 interrelated models can be written as:

\[
\begin{align*}
P_{d,1} &= f_1(P_{d-1,1}, P_{d-2,1}, \ldots, P_{d-1,24}, P_{d-2,24}, \ldots) + \varepsilon_{d,1} \rightarrow \hat{P}_{d,1}, \\
&\vdots \\
P_{d,24} &= f_{24}(P_{d-1,1}, P_{d-2,1}, \ldots, P_{d-1,24}, P_{d-2,24}, \ldots) + \varepsilon_{d,24} \rightarrow \hat{P}_{d,24},
\end{align*}
\]

which according to Chatfield (2000) and Diebold (2004) can be regraded as a multivariate model, since the dependency structure is interrelated.
It should be emphasized that both frameworks, defined by Eqns. (1) and (2), make explicitly (or implicitly) assumptions on the innovations for individual load periods. For instance, that for each hour \( \varepsilon_{d,h} \) follows a normal distribution with zero mean, i.e., \( \varepsilon_{d,h} \sim N(0, \sigma^2_h) \). However, they do not assume anything about the joint distribution of the innovations for different hours. To mitigate this unwanted feature, a fully multivariate modeling framework may be implemented which treats the price series as panel data:

\[
\begin{bmatrix}
P_{d,1} \\
P_{d,2} \\
\vdots \\
P_{d,24}
\end{bmatrix} = f\left(\begin{bmatrix}
P_{d-1,1} \\
P_{d-1,2} \\
\vdots \\
P_{d-1,24}
\end{bmatrix}, \begin{bmatrix}
P_{d-2,1} \\
P_{d-2,2} \\
\vdots \\
P_{d-2,24}
\end{bmatrix}, \ldots, \begin{bmatrix}
\varepsilon_{d,1} \\
\varepsilon_{d,2} \\
\vdots \\
\varepsilon_{d,24}
\end{bmatrix}\right) \rightarrow \begin{bmatrix}
\hat{P}_{d,1} \\
\hat{P}_{d,2} \\
\vdots \\
\hat{P}_{d,24}
\end{bmatrix}.
\]

The model structure may be identical to that in Eqns. (1)-(2), but it allows for a joint estimation for all load periods, e.g., via multivariate Least Squares, multivariate Yule-Walker equations (as in this paper) or maximum likelihood (L"utkepohl [2005]). Hence, there is an explicit (or implicit) joint distribution assumption on the error vector \( \varepsilon_d = [\varepsilon_{d,1}, \ldots, \varepsilon_{d,24}]' \), e.g., that \( \varepsilon_d \sim N_{24}(\mathbf{0}, \Sigma) \), where \( \Sigma \) is a 24-dimensional covariance matrix of a multivariate normal distribution. Comparing Eqn. (3) with (2) it is clear that the former is more general and the set of 24 interrelated models can be nested in the fully multivariate model. From the statistical point of view, Eqn. (2) describes only the marginal distribution of \( P_d \), but in an interrelated way.

Note, that it is also possible to estimate a fully multivariate model using a two-step procedure. First estimating separately the 24 rows of Eqn. (3), then estimating the noise terms, e.g., \( \varepsilon_d \sim N_{24}(\mathbf{0}, \Sigma) \), using the residuals of the 24 models. However, in such a case the dependencies between the 24 single model specifications are ignored and the estimation procedure may lead to a suboptimal in-sample fit. But not necessarily worse forecasts. In general, as Chatfield (2000) emphasizes, while fully multivariate models are usually found to yield a better in-sample fit, their forecasts need not necessarily be better.

Vector AutoRegression (VAR) is the basic modeling structure in the fully multivariate context, for sample EPF applications see Huisman et al. (2007), Panagiotelis and Smith (2008), Haldrup et al. (2010) and He et al. (2015). However, if the number of parameters is very large it may be a good idea to reduce dimensionality of the problem first and consider factor models, as in Garcia-Martos et al. (2012), Wu et al. (2013), Maciejowska and Weron (2015, 2016) and Raviv et al. (2015). In one of the few applications in the computational intelligence EPF literature, Yamin et al. (2004) use a neural network with 24 nodes in the output layer, hence consider a fully multivariate approach.

2.2. The univariate modeling framework

In the second stream of EPF literature, the day-ahead electricity prices are modeled by a univariate time series model for \( P_t \):

\[
P_t = f(P_{t-1,}, P_{t-2,}, \ldots) + \varepsilon_t,
\]

where \( \varepsilon_t \) is the innovation term at time \( t \), and \( f(\cdot) \) is some function of the explanatory variables. Similarly as in Eqns. (1)-(3), the univariate time series model for the hourly prices \( P_t \) makes either an explicit or an implicit assumption on the innovations, e.g., that \( \varepsilon_t \sim N(0, \sigma^2) \). However, in
contrast to the multivariate modeling framework, predicting the next day’s 24 hourly prices now involves computing one- to 24-step ahead forecasts:

- either in a recursive or iterative scheme (as in this paper), where the price forecast for hour \( t \) is used as an explanatory variable when forecasting the price for hour \( (t + 1) \), i.e.,
\[
\hat{P}_t = f(P_{t-1}, P_{t-2}, \ldots) \longrightarrow \hat{P}_{t+1} = f(\hat{P}_t, P_{t-1}, \ldots),
\]
- or directly by computing 24-step ahead forecasts for each load period:
\[
\hat{P}_t = f(P_{t-24}, P_{t-25}, \ldots),
\]
\[
\hat{P}_{t+1} = f(P_{t-23}, P_{t-24}, \ldots),
\]
\[
etc., \text{as in Keles et al. (2016); note, that this approach is a special case of the recursive scheme with } \hat{P}_t \text{ not depending on } P_{t-1}, \ldots, P_{t-23}.
\]

The drawback of the recursive scheme is that it is sensitive to the accumulation of errors because they propagate forward, while the latter method does not use the most recent information (which may decrease performance for the late night and early morning hours).

2.3. Frameworks vs. models

We consider two major modeling frameworks – multivariate and univariate. The multivariate framework uses an explicit ‘day × hour’, matrix-like structure for the 24-dimensional electricity price vector \( P_d \), as introduced in Eqns. (1)-(3), and either explicitly or implicitly assumes that the residual variance is different for each of the 24 (48 or more) load periods, i.e., \( \text{Var}(\epsilon_{d,h1}) \neq \text{Var}(\epsilon_{d,h2}) \). The 24 individual models can be either estimated independently or jointly in a fully multivariate framework, see Eqn. (3), while prices for all load periods of the next day are predicted at once as one-step (de facto one-day) ahead forecasts.

Note, that since the estimation in Eqn. (1) is conducted independently for each load period and not jointly for all, and there are no interdependencies between the models for individual load periods, many authors would not call such models multivariate. However, sets of estimated independently, but interdependent models, as defined by Eqn. (2), are often regarded as multivariate (Chatfield, 2000; Diebold, 2004). Finally, the framework defined in Eqn. (3) clearly leads to multivariate models (Lütkepohl, 2005).

The univariate framework treats prices as one ‘high-frequency’ hourly time series, as in Eqn. (4). The univariate models are estimated jointly for all load periods and either explicitly or implicitly assume that the residual variance, i.e., \( \text{Var}(\epsilon_t) \), is the same across all hours (unless an additional variance model is specified). Forecasting with these models requires either using a recursive scheme and computing a series of 24 one-step ahead forecasts or calculating 24-step ahead predictions for each load period.

2.4. Converting univariate to multivariate frameworks and vice versa

We should also note, that in general a univariate framework can be converted into a multivariate one and vice versa, similarly as \( P_{d,h} \) can be rewritten into \( P_t = P_{24d+h} \). However, when doing so the implicit (and partially the explicit) error structure changes. For instance, when changing a multivariate framework into a univariate one, an implicit assumption on innovations \( \epsilon_{d,h} \) turns into an implicit assumption on univariate innovations \( \epsilon_t \). Here, the innovation specification gets simplified, since \( \epsilon_{d,h} \) may have a different variance for each hour \( h \), but \( \epsilon_t \) has a constant variance for all load periods. If we have an explicit innovation specification within a multivariate framework,
e.g., \( e_d \sim N(0, \Sigma) \), then it is preserved on the marginal distribution level. Hence, the resulting innovation specification is \( e_t \sim N(0, \sigma_t^2) \) with \( \sigma_t^2 \) being the diagonal elements of \( \Sigma \).

On the other hand, when changing a univariate framework to a multivariate one, the implicit assumption that innovations \( e_t \) follow the same distribution, e.g., \( \text{Var}(e_t) = \sigma^2 \), turns to an implicit assumption that the innovations have a different distribution for every hour of the day, e.g., \( \text{Var}(e_{d,h}) = \sigma_h^2 \). If we have an explicit distribution assumption, e.g., \( e_t \sim N(0, \sigma^2) \), it turns into a multivariate distribution assumption, e.g., \( e_d \sim N(0, \Sigma) \) with a model specific restriction on the diagonal of \( \Sigma \). However, this representation is usually not unique as a multivariate distribution cannot be represented uniquely by its marginal distributions.

In practice, the forecasting impact of changing the model representation for implicit innovation specifications is marginal, since many estimation methods (e.g., OLS) are asymptotically consistent for homoscedastic but also for heteroscedastic innovation specifications. For illustration purposes, in [Appendix B] we show alternative representations of selected models.

3. Datasets and data preprocessing

3.1. Datasets

To conduct a thorough empirical study we consider a total of 12 electricity spot price datasets, see Table [1]. Note, that like Uniejewski et al. (2016), we use the terms spot and day-ahead interchangeably, which is line with the majority of literature on European electricity markets. However, in the U.S., the spot market is another name for the real-time market, while the day-ahead market is usually called the forward market (Burger et al., 2007; Weron and Ziel, 2018).

Eleven datasets come from six major European power markets, including the European Power Exchange (EPEX SPOT) for power spot trading in Germany, France, Austria, Switzerland and Luxembourg, the Nordic power exchange Nord Pool and OMIE, which manages the Iberian markets (Spain and Portugal). All eleven concern day-ahead markets with 24 hourly load periods and cover a six year period from 30 July 2010 to 28 July 2016. The last dataset comes from the price track of the Global Energy Forecasting Competition 2014 (GEFCom2014), the largest energy forecasting competition to date (Hong et al., 2016), and includes locational marginal prices (LMPs, i.e., zonal prices) at an hourly resolution from 1 January 2011 to 17 December 2013. The exact origin of the data has never been revealed by the organizers but – given its features – comes from one of the U.S. markets.

In the empirical analysis we use a 730-day (ca. two-year) rolling calibration window. First, all considered models are estimated using data from the initial calibration period (i.e., from 31 July 2010 to 30 July 2012 for the European datasets and from 1 January 2011 to 30 December 2012 for GEFCom2014) and forecasts for all 24 hours of the next day (respectively 31 July 2012 and 31 December 2012) are determined. Then the window is rolled forward by one day, the models are reestimated and forecasts for all 24 hours of the next day are computed. This procedure is repeated until the predictions for the 24 hours of the last day in the out-of-sample test period (respectively 28 July 2016 and 17 December 2013) are made. Note, that for the European datasets we are left with roughly four years (1459 days) of data for out-of-sample testing and for the GEFCom2014 dataset with only 352 days. Note also, that because of the clock-change issue, we have to do minor adjustments to the data to obtain well defined price processes. We interpolate the missing hour in
Table 1: Summary table of the considered electricity spot price series, i.e., day-ahead prices at hourly resolution. The GEFCom2014 dataset covers a three year period from 1 January 2011 to 17 December 2013, the remaining datasets – a six year period from 30 July 2010 to 28 July 2016.

<table>
<thead>
<tr>
<th>Electricity market and region</th>
<th>Acronym</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELPEX price for Belgium</td>
<td>BELPEX.BE</td>
<td>EUR/MWh</td>
<td>belpex.be</td>
</tr>
<tr>
<td>EPEX price for Switzerland</td>
<td>EPEX.CH</td>
<td>EUR/MWh</td>
<td>epexspot.com</td>
</tr>
<tr>
<td>EPEX price for Germany and Austria</td>
<td>EPEX.DE+AT</td>
<td>EUR/MWh</td>
<td>epexspot.com</td>
</tr>
<tr>
<td>EPEX price for France</td>
<td>EPEX.FR</td>
<td>EUR/MWh</td>
<td>epexspot.com</td>
</tr>
<tr>
<td>EXAA price for Germany and Austria</td>
<td>EXAA.DE+AT</td>
<td>EUR/MWh</td>
<td>exaa.at</td>
</tr>
<tr>
<td>GEFCom2014 competition data</td>
<td>GEFCOM2014</td>
<td>USD/MWh</td>
<td>Hong et al. (2016)</td>
</tr>
<tr>
<td>Nord Pool price for West Denmark</td>
<td>NP.DK1</td>
<td>EUR/MWh</td>
<td>nordpoolspot.com</td>
</tr>
<tr>
<td>Nord Pool price for East Denmark</td>
<td>NP.DK2</td>
<td>EUR/MWh</td>
<td>nordpoolspot.com</td>
</tr>
<tr>
<td>Nord Pool System price</td>
<td>NP.SYS</td>
<td>EUR/MWh</td>
<td>nordpoolspot.com</td>
</tr>
<tr>
<td>OMIE price for Spain</td>
<td>OMIE.ES</td>
<td>EUR/MWh</td>
<td>omie.es</td>
</tr>
<tr>
<td>OMIE price for Portugal</td>
<td>OMIE.PT</td>
<td>EUR/MWh</td>
<td>omie.es</td>
</tr>
<tr>
<td>OTE price for the Czech Republic</td>
<td>OTE.CZ</td>
<td>EUR/MWh</td>
<td>ote-cr.cz</td>
</tr>
</tbody>
</table>

March and average the doubled hour in October for the European data. The GEFCom2014 data was released clock-change adjusted, however, the used adjustment methodology is not reported in [Hong et al. (2016)].

3.2. Variance stabilizing transformation

It is widely known that many electricity price series exhibit price spikes, mostly positive but in some markets also negative [Fanone et al. 2013; Nowotarski et al. 2014]. For electricity prices with only positive values the logarithmic transform is very popular to reduce spike severity and consequently stabilize the variance. However, for datasets with very close to zero or negative prices the log-transform is not feasible. In such cases, typically no transformation is used. This is reasonable for moderately spiky data like the German/Austrian EPEX prices. But for datasets with extreme spikes, like the French EPEX prices, such a ‘raw data approach’ requires robust estimation algorithms (see e.g. [Huber and Ronchetti 2009] or models with embedded spike components (like in [Weron 2009]). However, such robust techniques are not popular in EPF and most studies utilize standard least-squares methods.

As we want to conduct a comprehensive forecasting study involving many diverse datasets, we have to deal with this problem in an automated way. The time series forecasting literature usually suggests the Box-Cox transform [Hyndman and Athanasopoulos 2013]. However, it has the disadvantage of returning a bi-modal marginal distribution of the transformed prices. As a viable alternative, we propose the area (or inverse) hyperbolic sine transformation (see [Uniejewski et al. 2017] for a recent review of variance stabilizing transformations):

\[
\text{asinh}(x) = \log \left( x + \sqrt{x^2 + 1} \right),
\]

for standardized spot prices \( x = \frac{1}{b} \{ P_{d,h} - a \} \). In Figure 1 the original and transformed electricity prices are visualized for two series that exhibit positive and negative price spikes. We can see that the
Figure 1: EPEX spot price $P_{d,h}$ in EUR/MWh for Germany and Austria (EPEX.DE+AT; top row) and France (EPEX.FR; third row), and the asinh-transformed prices, i.e., $Y_{d,h}$, for both markets (second and bottom rows). The marginal densities are depicted in the right panels.
spikes become less severe, but do not vanish completely. The area hyperbolic sine transformation has been originally used in the EPF context by Schneider (2011), but the article went unnoticed.

In our empirical study in Section 5 we use the (median, MAD) normalization, i.e., we set the shift parameter, $a$, equal to the median of the 730-day calibration sample and the scale parameter, $b$, equal to the sample median absolute deviation (MAD) around the sample median adjusted by a factor for asymptotically normal consistency to the standard deviation (see Uniejewski et al., 2017). This factor is $\frac{1}{z_{0.75}} \approx 1.4826$ where $z_{0.75}$ is the 75% quantile of the normal distribution; in R this is the default option if one runs mad(x), in Matlab this corresponds to $1.4826 \times \text{mad}(x, 1)$. The transform acts so that close to $a$ the transformation is almost linear, whereas positive and negative price spikes are pulled towards the center in a logarithmic way; asymptotically $\text{asinh}(x) \approx \text{sign}(x) \log(2|x|)$ as $|x| \to \infty$.

In what follows, we denote by $Y_{d,h}$ (or $Y_t$ in the univariate context) the transformed data, i.e.,

$$Y_{d,h} = \text{asinh} \left( \frac{P_{d,h} - a}{b} \right)$$

and calibrate all models to the asinh-transformed prices (except for the naive model defined in Section 4.2). Once the forecasts $\hat{Y}_{d,h}$ are computed we apply the inverse transform, i.e., the hyperbolic sine, to obtain the day-ahead electricity price forecasts:

$$\hat{P}_{d,h} = b \cdot \sinh \left( \hat{Y}_{d,h} \right) + a$$

and use the latter to evaluate and compare the models in Section 5.

4. Models

Our choice of the forecasting models is guided by the existing literature on short-term EPF and the desire to perform a comprehensive study that addresses the three pertinent questions put forward in the Introduction. As we want to focus on the explanatory power of the past spot prices, we consider 'pure price' or 'price only' models, i.e., models without exogenous (stochastic) variables, like weather, load or renewable energy generation forecasts.

Overall we consider 58 models from eight classes, see Appendix A. However, for clarity of exposition, in the main body of the text we focus only on 10 models – one best performing model from each of the eight classes and the second best performer from the two best classes (i.e., C6 and C8):

C1. the weekly mean of hourly frequency benchmark, which is a simple periodic function denoted by meanHoW,
C2. the so-called naive benchmark of Nogales et al. (2002), which belongs to the class of similar-day techniques denoted by naive,

---

1Strictly speaking, however, our models include also other non-price variables – dummies representing calendar effects. Yet, as is common in the EPF literature (Weron, 2014), we do not treat them as exogenous variables since their nature is deterministic and they can be removed prior to fitting a stochastic model to prices, like in the 24AR and AR-type models considered here.
C3. 16 parsimonious autoregressive (AR) models within a multivariate framework, built on some prior knowledge of experts and following Uniejewski et al. (2016) and Ziel (2016a) called expert models or experts, estimated using Ordinary Least Squares (OLS) \( \rightarrow \) represented by \( \text{expert}_{\text{DoW}, \text{nl}} \).

C4. two AR specifications composed of sets of 24 independent models (for each hour of the day) and estimated using Yule-Walker equations \( \rightarrow \) represented by \( 24\text{AR}_{\text{HoW}} \).

C5. two vector autoregressive (VAR) models estimated using multivariate Yule-Walker equations, i.e., the only fully multivariate models in this study \( \rightarrow \) represented by \( \text{VAR}_{\text{HoW}} \).

C6. 16 parameter-rich AR structures within a multivariate framework, estimated using the least absolute shrinkage and selection operator (i.e., lasso or LASSO), which shrinks to zero the coefficients of redundant explanatory variables \( \rightarrow \) represented by \( 24\text{lasso}^{\text{HQC}}_{\text{DoW}, \text{p,nl}} \) and \( 24\text{lasso}^{\text{HQC}}_{\text{DoW}, \text{nl}} \).

C7. four univariate AR models estimated using Yule-Walker equations \( \rightarrow \) represented by \( \text{AR}_{\text{HoW}} \).

C8. 16 univariate parameter-rich AR specifications estimated using the LASSO \( \rightarrow \) represented by \( \text{lasso}^{\text{HQC}}_{\text{HoW}, \text{p}} \) and \( \text{lasso}^{\text{HQC}}_{\text{HoW}} \).

In Section 4.1 we define the seasonal dummies and means used later in the text. Next in Sections 4.2-4.4 we describe the 10 representative models, starting with the two benchmarks, then moving on to autoregressive structures considered within a multivariate framework and concluding with univariate specifications. Model definitions for the remaining models can be found in Appendix A while a summary of results in Appendix C.

4.1. Seasonal dummies and means

Before we introduce the forecasting models let us briefly define three types of dummy variables and the corresponding (time-varying) means. Since the daily and weekly seasonalties are the most pronounced for electricity prices we define:

- the hour-of-the-day dummy for \( k = 1, \ldots, 24 \) and arbitrary \( d \):
  \[
  \text{HoD}^k_{d,h} = \begin{cases} 
  1 & \text{if } h \text{ is the } k\text{-th hour of the day,} \\
  0 & \text{otherwise},
  \end{cases}
  \]

- the day-of-the-week dummy for \( k = 1 \) (Monday), \( \ldots, 7 \) (Sunday) and arbitrary \( h \):
  \[
  \text{DoW}^k_{d,h} = \begin{cases} 
  1 & \text{if } d \text{ is the } k\text{-th day of the week,} \\
  0 & \text{otherwise},
  \end{cases}
  \]

- the hour-of-the-week dummy for \( k = 1 \) (Monday, hour 1), \( \ldots, 168 \) (Sunday, hour 24):
  \[
  \text{HoW}^k_{d,h} = \begin{cases} 
  1 & \text{if } 24(d-1) + h \text{ is the } k\text{-th hour of the week,} \\
  0 & \text{otherwise}.
  \end{cases}
  \]
In the univariate context we use the single time index notation \((\text{HoD}^k, \text{DoW}^k, \text{HoW}^k)\) and sometimes we omit the time index at all. Note, that it always holds: \(\text{HoW}^{k+24(k-1)} = \text{DoW}^k \text{HoD}^j\). Note also, that we implicitly assume that the data resolution is hourly, as for the 12 datasets described in Section 3. However, all methods considered here can be easily modified to work with half-hourly, 15 minute or even shorter load periods.

Given the three dummies – \(\text{HoW}^k, \text{HoD}^k\) and \(\text{DoW}^k\) – we introduce three time-varying means estimated with OLS within a standard linear regression framework:

- \(\bar{Y}_{\text{HoW},d,h}\) – the weekly mean of hourly frequency,
- \(\bar{Y}_{\text{HoD},d,h}\) – the daily mean of hourly frequency,
- \(\bar{Y}_{\text{DoW},d,h}\) – the weekly mean of daily frequency.

In the same manner, we introduce the sample mean of the full sample, \(\bar{Y}\). In the empirical study of Section 5 all four means are computed iteratively, for each 730-day calibration window. However, in Figure 2 we illustrate the four means using the whole 6-year sample of the EPEX spot price in Germany and Austria (EPEX.DE+AT).

### 4.2. Two simple benchmarks

The first benchmark, denoted by mean\(_{\text{HoW}}\), is given by the time-varying, i.e., estimated for each calibration window, weekly mean of hourly frequency of the asinh-transformed price \(Y_{d,h}\):

\[
Y_{d,h} = \bar{Y}_{\text{HoW},d,h} + \varepsilon_{d,h} = \sum_{k=1}^{168} \beta_k \text{HoW}^k_{d,h} + \varepsilon_{d,h}.
\]
Unless stated otherwise, the error terms ($\varepsilon_d, h$ or $\varepsilon_t$) in all models considered in this study are assumed to have zero mean, finite variance and are uncorrelated. Note, that we have also considered two other time-varying means defined in Section 4.1, $\overline{Y}_{\text{HoD},d,h}$ and $\overline{Y}_{\text{DoW},d,h}$, but their predictive performance was worse and is not reported in this study.

The second benchmark belongs to the class of similar-day techniques (for a taxonomy of EPF approaches see e.g. Weron, 2014):

$$P_{d,h} = \begin{cases} P_{d-7,h} + \varepsilon_{d,h}, & \text{DoW}_{d,h}^k = 1 \text{ for } k = 1, 6, 7, \\ P_{d-1,h} + \varepsilon_{d,h}, & \text{otherwise}, \end{cases}$$

and denote it by naive. Most likely it has been introduced to the EPF literature by Nogales et al. (2002). It proceeds as follows: the forecast for hour $h$ on Monday, Saturday and Sunday is set equal to the price for the same hour a week ago; the forecast for hour $h$ on the remaining days is set equal to the price for the same hour yesterday. As was argued by Nogales et al. (2002) and Conejo et al. (2005), forecasting procedures that are not calibrated carefully fail to outperform the naive method surprisingly often. Note, that we have also considered two simpler similar-day techniques: (i) $P_{d,h} = P_{d-1,h} + \varepsilon_{d,h}$ and (ii) $P_{d,h} = P_{d-7,h} + \varepsilon_{d,h}$, but their predictive performance was worse and is not reported in this study.

4.3. The multivariate modeling framework

Having discussed two simple benchmarks, we are now ready to define models considered in this study within the multivariate framework. They all use an explicit ‘day $\times$ hour’, matrix-like structure, implicitly assume that the residual variance is different for each load period, and are estimated:

- independently for each load period and do not admit interdependencies between the hours of the day, see Eqn. (1),
- independently for each load period but admit interdependencies between the hours of the day, see Eqn. (2),
- or jointly in a fully multivariate framework, see Eqn. (3).

In contrast to the univariate models discussed in Section 4.4, here the prices for all load periods of the next day are predicted at once as one-step (i.e, one-day) ahead forecasts.

We should note, that recently reported empirical evidence provides a fundamental justification of the multivariate framework, as opposed to the univariate. Namely, analyzing the bidding behavior in the Italian power market, Gianfreda et al. (2016) find that since the solar production suddenly decreases in the evening and the merit order curve rapidly shifts, the thermal and hydro producers are able to exert market power at this time of the day. They further speculate that these generators apply different bidding strategies for different hours, hence significantly change the respective price formation mechanisms.
4.3.1. An expert model

This class of models is based on a parsimonious autoregressive structure originally proposed by Misiorek et al. (2006) and later used in a number of EPF studies (Weron and Misiorek, 2008; Serinaldi, 2011; Kristiansen, 2012; Nowotarski et al., 2014; Gaillard et al., 2016; Maciejowska et al., 2016; Marcjasz et al., 2018; Maciejowska et al., 2016; Marcjasz et al., 2018; Nowotarski and Weron, 2018; Uniejewski et al., 2016; Ziel, 2016a). Since these models are built on some prior knowledge of experts, following Uniejewski et al. (2016) and Ziel (2016a), we refer to them as expert models. In the empirical comparison in Section 5, this class is represented by the expertDoW,nl model. Within this autoregressive structure the asinh-transformed price on day \( d \) and hour \( h \) is given by the following formula:

\[
Y_{d,h} = \beta_{h,1} + \beta_{h,2}Y_{d-1,h} + \beta_{h,3}Y_{d-2,h} + \beta_{h,4}Y_{d-7,h} + \beta_{h,5}Y_{d-1,\text{min}} + \beta_{h,6}Y_{d-1,\text{max}} + \beta_{h,7}Y_{d-1,24} + \sum_{j=1}^{7} \beta_{h,7+j}\text{DoW}_{d,h}^j + \epsilon_{d,h},
\]

(10)

where

- the lagged prices \( Y_{d-1,h}, Y_{d-2,h} \) and \( Y_{d-7,h} \) account for the autoregressive effects of the previous days (the same hour yesterday, two days ago and one week ago);
- \( Y_{d-1,\text{min}} = \min_{h=1,...,24}\{Y_{d-1,h}\} \) and \( Y_{d-1,\text{max}} = \max_{h=1,...,24}\{Y_{d-1,h}\} \) are respectively the minimum and the maximum of the previous day’s 24 hourly prices and create a link with all yesterday’s prices, not just the prices for the same hour; note, that the minimum and maximum are non-linear due to \( \min(x,y) = 0.5(x + y - |x - y|) \) and \( \max(x,y) = 0.5(x + y + |x - y|) \), hence subscript nl in the model name;
- \( Y_{d-1,24} \) is the price for the last load period of the previous day and is included in (10) to take advantage of the fact that prices for early morning hours depend more on the previous day’s price at midnight than on the price for the same hour, as recently emphasized by Maciejowska and Nowotarski (2016) and Ziel (2016a);
- and DoW_{d,h}^i, \( i = 1, ..., 7 \) are the daily dummies, hence subscript DoW in the model name.

Note, that if \( h = 24 \) then the term which includes \( \beta_{h,7} \) is collinear with \( \beta_{h,2} \). Hence, in this case, the model has fewer parameters. We estimate the parameters using OLS.

The expertDoW,nl model is inspired by the mAR1hm and AR2hm models of Uniejewski et al. (2016), but does not include a dummy for public holidays (for the sake of parsimony) and a term that depends on the average price of the previous day. The impact of the latter, however, has been shown by Uniejewski et al. (2016) to be negligible, hence the change.

4.3.2. A set of 24 AR models

This class of benchmarks is very popular in the EPF literature. In our setup, the demeaned with respect to \( \bar{Y}_{\text{HoW},d,h} \) asinh-transformed price is modeled as a standard autoregressive process
of order \( p_h \), i.e., AR(\( p_h \)), independently for each hour \( h \). Formally, the model – denoted later in the text by \( 24\text{AR}_{\text{HoW}} \) – is given by:

\[
Y_{d,h} = \bar{Y}_{\text{HoW},d,h} + \phi_{0,h} + \sum_{k=1}^{p_h} \phi_{k,h}(Y_{d-k,h} - \bar{Y}_{\text{HoW},d,h}) + \varepsilon_{d,h},
\]  

(11)

where \( \phi_{k,h} \) are the autoregressive parameters. We estimate the model by solving the Yule-Walker equations with a maximum order \( p_{h,\text{max}} = 8 \), to cover a potential dependency of up to 8 days. Each \( p_h \) is chosen based on the Akaike Information Criterion (AIC), see e.g. [Hyndman and Athanasopoulos (2013) or Ziel and Steinert (2016)].

In contrast to the expert model defined in Section 4.3.1, the \( 24\text{AR}_{\text{HoW}} \) model does not admit any interdependencies between the prices for different load periods. Although it is written in a multivariate framework, it actually is a set 24 independent univariate models at daily frequency, one for each hour, like in Eqn. (1). Note also, that this model is a special case of a 24-dimensional Vector AutoRegressive (VAR) model with diagonal parameter matrices, see Section 4.3.3 below.

### 4.3.3. A VAR model

Using matrix notation we introduce a Vector AutoRegressive (VAR) structure, denoted later in the text by \( \text{VAR}_{\text{HoW}} \):

\[
Y_d = \bar{Y}_{\text{HoW},d} + \phi_0 + \sum_{k=1}^{p} \Phi_k(Y_{d-k} - \bar{Y}_{\text{HoW},d}) + \varepsilon_d,
\]  

(12)

where \( Y_d = [Y_{d,1}, \ldots, Y_{d,24}] \) with its mean vector \( \bar{Y}_{\text{HoW},d} \) across all available days in the calibration sample (which corresponds to the 168 possible values of \( \bar{Y}_{\text{HoW},d,h} \), 24 for each of the seven days of the week). \( \Phi_k \) is a parameter matrix, \( \phi_0 \) is the intercept vector and \( \varepsilon_d = [\varepsilon_{d,1}, \ldots, \varepsilon_{d,24}] \). We calibrate the model by solving the multivariate Yule-Walker equations, see e.g. [Lütkepohl (2005)], with \( p_{\text{max}} = 8 \) to cover the same memory as for the model in Section 4.3.2.

### 4.3.4. Multivariate lasso models

As has been noted in a number of studies, both statistical and computational intelligence, a key point in EPF is the appropriate choice of explanatory variables ([Amjadi and Keynia, 2009], [Gianfreda and Grossi, 2012], [González et al., 2015], [Karakatsani and Bunn, 2008], [Keles et al., 2016], [Maciejowska, 2014], [Voronin and Partanen, 2013], [Weron, 2014]). The typical approach has been to select predictors in an ad hoc fashion, sometimes using expert knowledge, seldom based on formal selection or shrinkage procedures for high-dimensional model specifications (like in [Gaillard et al., 2016], [Ludwig et al., 2015], [Uniejewski et al., 2016], [Ziel, 2016a], [Ziel et al., 2015a]).

Recall, that shrinkage (also known as regularization) fits the full model with all predictors using an algorithm that shrinks coefficients of the less important explanatory variables towards zero ([James et al., 2013]). Some shrinkage methods, like the least absolute shrinkage and selection operator (i.e., lasso or LASSO) introduced by [Tibshirani (1996)], may actually shrink some of the coefficients to zero itself, thus de facto performing variable selection. It should be noted, however, that while variable selection is beneficial for interpretability, for reducing the forecasting errors only the shrinkage property is crucial ([Uniejewski et al., 2016]).
Let us first introduce a general regression model, somewhat inspired by the full ARX or fARX model of [Uniejewski et al. (2016)], and then define two special cases considered later in Section 5 (the remaining models from this class are defined in Appendix A.4). For each hour of the day, \( h = 1, \ldots, 24 \), the model is given by:

\[
Y_{d,h} = \sum_{l=1}^{24} \sum_{k=1}^{8} \phi_{h,k,l,0} Y_{d-l,k} + \sum_{k=1}^{8} \phi_{h,k,\min,0} Y_{d-k,\min} + \sum_{k=1}^{8} \phi_{h,k,\max,0} Y_{d-k,\max} + \sum_{j=1}^{7} \phi_{h,0,0,j} \text{DoW}_{d,h}^j + \sum_{j=1}^{7} \phi_{h,1,1,j} \text{DoW}_{d,h}^j Y_{d-1,h} + \sum_{j=1}^{7} \phi_{h,1,24,j} \text{DoW}_{d,h}^j Y_{d-1,24} + \epsilon_{d,h}. \tag{13}
\]

The first term describes the autoregressive effects up to eight days ago, the second and third terms specify the lagged non-linear effects using the minimum and maximum price of the day (again up to eight days ago), the fourth term describes the day-of-the-week effect and the fifth and sixth terms are the periodic effects (as seen in the periodic expert models, see Section 4.3.1). Note, that the \( \phi \)'s are indexed by four variables, with the first one indicating the target hour (i.e., \( h \)). The remaining three refer to the time lag in days (\( k = 1, \ldots, 8 \) or 0), the hour of the day (\( l = 1, \ldots, 24 \) or an aggregate value for all 24 hours of the day (\( l = \text{min}, \text{max} \)), and the day of the week (\( j = 1, \ldots, 7 \) or 0).

We denote this model by \( 24\text{lasso}_{\text{DoW,p,all}} \), because it is embedded in a multivariate framework (one equation for each of the 24 hours), estimated via the lasso (independently for each load period, see below), with day-of-the-week, periodic and non-linear effects. The superscript HQC denotes the Hannan-Quinn Information Criterion used to select the tuning (or regularization) parameter \( \lambda \) within the calibration window, see Appendix A.4 for details. Based on Eqn. (13), we introduce a variant without the periodic parameter terms (i.e., with \( \phi_{h,1,1,j} = \phi_{h,1,24,j} = 0 \) and denote it by \( 24\text{lasso}_{\text{DoW,p}} \).

Now, let us comment on the calibration procedure. For this purpose, let us write Eqn. (13) in a more compact form:

\[
Y_{d,h} = X_{d,h}' \hat{\beta}_h + \epsilon_{d,h}, \tag{14}
\]

where \( X_{d,h} \) is a vector of the regressors and \( \hat{\beta}_h \) is a vector of their coefficients. Given a \( D = 730 \) observation sample \( Y_h = [Y_{1,h}, \ldots, Y_{D,h}]' \), the sample representation is as follows:

\[
Y_h = X_h' \hat{\beta}_h + \epsilon_h, \tag{15}
\]

where \( X_h = [X_{1,h}', \ldots, X_{D,h}']' \) and \( \epsilon_h = [\epsilon_{1,h}, \ldots, \epsilon_{D,h}]' \). To efficiently estimate the model using the lasso, we require a scaled version of the OLS equation, i.e.:

\[
\tilde{Y}_h = \tilde{X}_h \tilde{\beta}_h + \tilde{\epsilon}_h, \tag{16}
\]

where \( \tilde{Y}_h \) and \( \tilde{X}_h \) are the scaled versions of \( Y_h \) and \( X_h \), respectively, i.e. the \( \| \cdot \|_2 \)-norm of each column is 1. Then the lasso estimator is given by [Hastie et al. 2015]:

\[
\hat{\beta}_{h,1} = \arg \min_{\beta} \| \tilde{Y}_h - \tilde{X}_h' \beta \|^2_2 + \lambda \| \beta \|_1, \tag{17}
\]
with the tuning (or regularization) parameter \( \lambda \geq 0 \). For \( \lambda = 0 \) we receive the OLS estimator of \( \tilde{\beta}_h \). Given \( \tilde{\beta}_{h,\lambda} \), by rescaling we can easily compute \( \hat{\beta}_h \).

4.4. The univariate modeling framework

Now, let us turn to the univariate framework and consider models for which the forecasts are computed recursively – the price forecast for hour 1 is used as input (i.e., an explanatory variable) when making the prediction for hour 2, etc. We will start with relatively standard AR model, then continue with parameter rich structures estimated via the lasso.

4.4.1. A univariate AR model

This univariate model, denoted later in the text by \( \text{AR}_{\text{HoW}} \), is a counterpart of the \( 24\text{AR}_{\text{HoW}} \) model defined in Section 4.3.2:

\[
Y_t = Y_{\text{HoW},t} + \phi_0 + \sum_{k=1}^{p} \phi_k (Y_{t-k} - Y_{\text{HoW},t}) + \epsilon_t,
\]

where \( Y_{\text{HoW},t} \) is the hour-of-the-week mean in the calibration period, \( \phi_k \) are the autoregressive parameters and \( p \) is the order of the AR process. Again, we estimate the model by solving the Yule-Walker equations and minimizing the AIC with respect to the maximum considered AR order \( p_{\text{max}} = 196 \). Note, that \( p_{\text{AR}} = 196 \) hours corresponds to \( p_{24\text{AR}} = 8 \) days, i.e., a potential memory of eight days. Interestingly, in Ziel et al. (2015a), \( \text{AR}_{\text{HoW}} \) served as a very strong benchmark for the EPEX.DE+AT market and outperformed a number of sophisticated model structures.

4.4.2. Univariate lasso models

Like \( \text{AR}_{\text{HoW}} \) is a counterpart of \( 24\text{AR}_{\text{HoW}} \), the two univariate models considered here are similar to the multivariate lasso models discussed in Section 4.3.4. The \( \text{lasso}^\text{HQC}_{\text{HoW},p} \) model is given by:

\[
Y_t = \sum_{k=1}^{168} \phi_{0,k} \text{HoW}_t^k + \sum_{k=1}^{196} \phi_{1,k} Y_{t-k} + \sum_{k=1}^{168} \phi_{2,k} \text{HoW}_t^k Y_{t-1} + \sum_{k=1}^{168} \phi_{3,k} \text{HoW}_t^k Y_{t-24} + \epsilon_t,
\]

where superscript \( \text{HQC} \) denotes the information criterion used for the lasso estimation method. A variant without periodic effects (i.e., with \( \phi_{2,k} = \phi_{3,k} = 0 \)) is denoted by \( \text{lasso}^\text{HQC}_{\text{HoW}} \). Note, that the \( \text{AR}_{\text{HoW}} \) model defined in Section 4.4.1 has the same structure as \( \text{lasso}^\text{HQC}_{\text{HoW}} \), but is estimated in a different way.

5. Empirical results

5.1. Performance evaluation in terms of MAE

For each of the eleven European datasets we have \( D = 1459 \) days (or approximately 4 years) in the out-of-sample test period. For the GEFCOM2014 only about 1 year (exactly \( D = 352 \) days) of out-of-sample data is available, however, given its popularity (gained during the competition),
Table 2: Mean Absolute Errors (MAE) in EUR/MWh (or USD/MWh for GEFCom2014) for the full out-of-sample period, as defined by Eqn. (20), for the 10 models defined in Section 4 and all 12 datasets. A heat map is used to indicate better (→ green) and worse (→ red) performing models. In the bottom rows we report the mean percentage deviation from the best (m.p.d.f.b.) model, as defined by Eqn. (21), for the MAE and the Root Mean Square Error (RMSE; the individual RMSE’s are not reported due to space limitations, but are available from the authors upon request).

<table>
<thead>
<tr>
<th>Market</th>
<th>mean$_{HoW}$</th>
<th>naive</th>
<th>expert$_{DoW,na}$</th>
<th>24AR$_{HoW}$</th>
<th>VAR$_{HoW}$</th>
<th>24lasso$_{HoW}$</th>
<th>HQC$_{DoW,na}$</th>
<th>HQC$_{24lasso_HQC}$</th>
<th>AR$_{HoW}$</th>
<th>HQC$_{lasso_HQC}$</th>
<th>HQC$_{lasso_HQC}$</th>
<th>m.p.d.f.b (MAE) (%)</th>
<th>m.p.d.f.b (RMSE) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPEX.CH</td>
<td>9.909</td>
<td>5.716</td>
<td>4.072</td>
<td>4.300</td>
<td>4.139</td>
<td>3.945</td>
<td>3.926</td>
<td>4.035</td>
<td>3.993</td>
<td>4.013</td>
<td>4.013</td>
<td>47.60</td>
<td>47.93</td>
</tr>
<tr>
<td>EPEX.DE+AT</td>
<td>8.110</td>
<td>7.751</td>
<td>5.225</td>
<td>5.636</td>
<td>5.200</td>
<td>5.105</td>
<td>5.118</td>
<td>5.078</td>
<td>5.058</td>
<td>5.048</td>
<td>5.048</td>
<td>3.36</td>
<td>2.61</td>
</tr>
<tr>
<td>NP.DK1</td>
<td>8.059</td>
<td>7.340</td>
<td>5.191</td>
<td>5.668</td>
<td>5.248</td>
<td>5.106</td>
<td>5.128</td>
<td>5.151</td>
<td>5.086</td>
<td>5.079</td>
<td>5.079</td>
<td>0.58</td>
<td>0.79</td>
</tr>
<tr>
<td>NP.SYS</td>
<td>6.441</td>
<td>2.680</td>
<td>1.806</td>
<td>1.975</td>
<td>2.062</td>
<td>1.686</td>
<td>1.692</td>
<td>1.783</td>
<td>1.763</td>
<td>1.752</td>
<td>1.752</td>
<td>6.02</td>
<td>5.36</td>
</tr>
</tbody>
</table>

As the main evaluation criterion we consider the Mean Absolute Error (MAE) for the full out-of-sample period. It is computed for each model and dataset as:

$$\text{MAE} = \frac{1}{24D} \sum_{d=1}^{D} \sum_{h=1}^{24} |\hat{\varepsilon}_{d,h}|,$$  \hspace{1cm} (20)

where $\hat{\varepsilon}_{d,h}$ denotes the estimated forecasting error for day $d$ and hour $h$. The MAE errors for the 10 models defined in Section 4 are reported for all 12 datasets in Table 2.\footnote{The MAE errors for all 58 models and four major markets are visualized in Figure C.6 in Appendix C.} We have also analyzed Root Mean Square Errors (RMSE), but the results were qualitatively the same and, hence, are not reported nor analyzed here due to space limitations (but are available from the authors upon request). Only in Table 2 we provide for comparison an aggregate measure of fit based on the RMSE – the m.p.d.f.b. – as defined in (21) below. Clearly, the relative forecasting performance of the models and their ranking is nearly identical, irrespective of whether MAE or RMSE is used.

[17]
In Table 2, we can clearly see the dominance of the lasso models over the competitors. However, there is no single lasso model that is the best for all datasets. The multivariate \( \text{24lasso}^{\text{HQC}}_{\text{DoW,al}} \) and \( \text{24lasso}^{\text{HQC}}_{\text{DoW,p,al}} \) models are the best for seven datasets: BELPEX.BE, EPEX.CH, EPEX.FR, GEFCom2014, NP.SYS, OMIE.ES and OMIE.PT, while the univariate \( \text{lasso}^{\text{HQC}}_{\text{DoW,p}} \) model is the best for the remaining five datasets: EPEX.DE+AT, EXAA.DE+AT, NP.DK1, NP.DK2 and OTE.CZ.

Given the full set of results for all 58 models (see Appendix A) and all 12 datasets it is hard to rank the models. In particular, it is not possible to make conclusive statements about the outperformance of the univariate modeling framework by the multivariate one or vice versa. To tackle this issue we introduce the mean percentage deviation from the best (m.p.d.f.b.) model, which is inspired by the m.d.f.b. measure used by Weron and Misiorek (2008) and Nowotarski et al. (2014), among others. The m.p.d.f.b. measure for model \( i \) indicates how similar is this model’s performance to the ‘optimal model’ composed of the best performing model for each of the 12 datasets:

\[
\text{m.p.d.f.b.} = \frac{1}{12} \sum_{j=1}^{12} \left| \frac{\text{ERR}_{i,j} - \text{ERR}_{\text{best model},j}}{\text{ERR}_{\text{best model},j}} \right| \times 100%,
\]

where \( \text{ERR}_{\text{best model},j} = \min_{1 \leq i \leq 58} \text{ERR}_{i,j} \) and \( \text{ERR} \) can be the MAE, as defined in Eqn. (20), the Root Mean Square Error (RMSE) or any other error measure for point forecasts.

The m.p.d.f.b. measures are reported in the bottom rows of Table 2. Somewhat surprisingly, we find that the relatively simple, univariate \( \text{AR}^{\text{HoW}}_\text{al} \) model with a m.p.d.f.b. \( \text{MAE} \) of 1.90% is only slightly worse than the more sophisticated lasso structures. For two markets (EPEX.DE+AT and OTE.CZ) it even beats the overall best performing \( \text{24lasso}^{\text{HQC}}_{\text{DoW,p,al}} \) model. This confirms the findings of Ziel et al. (2015a) who found this model to be a very strong benchmark for the EPEX.DE+AT market, that outperformed a number of sophisticated model structures.

The next in terms of m.p.d.f.b. are the \( \text{expert}^{\text{DoW,al}} \) and \( \text{expert}^{\text{DoW,p,al}} \) models with nearly identical performance (on average, but not across all datasets). They are extremely competitive for the Iberian markets. In particular, \( \text{expert}^{\text{DoW,p,al}} \) is second best in terms of MAE (only after \( \text{24lasso}^{\text{HQC}}_{\text{DoW,p,al}} \)) for the OMIE.PT dataset.

The performance of the AR and VAR-type structures considered within a multivariate framework is rather disappointing. The \( \text{24AR}^{\text{HoW}}_\text{al} \) model is worse than all expert specifications and \( \text{VAR}^{\text{HoW}}_\text{al} \) is not much better (see Appendix C for details). At the very end, which is not surprising, are the simple benchmarks – \( \text{mean}^{\text{HoW}} \) and \( \text{naive} \) are significantly (see also the discussion in Section 5.2) outperformed by the more sophisticated models across all hours, seasons of the year and markets.

Finally, let us look at model performance in the four seasons of the year: Spring (March, April, May), Summer (June, July, August), Fall (September, October, November) and Winter (December, January, February). Of course, there is some variability in forecasting accuracy across the seasons. For instance, in the Summer, \( \text{expert}^{\text{DoW,al}} \) is the best performing model for both Iberian datasets, but \( \text{lasso}^{\text{HQC}}_{\text{HoW}} \) and \( \text{24lasso}^{\text{HQC}}_{\text{DoW,al}} \) follow closely by. In the Fall, \( \text{AR}^{\text{HoW}}_\text{al} \) is the best for EPEX.DE+AT and – somewhat surprisingly – the best overall in terms of m.p.d.f.b. MAE (see Table 2), but the

\[3\] The m.p.d.f.b. measure for all 58 models is plotted in Fig. C.7 in Appendix C.
Table 3: Mean percentage deviation from the best model for the MAE errors, i.e., m.p.d.f.b\_MAE \,(in \,\%\,), in the four seasons of the year: Spring (March, April, May), Summer (June, July, August), Fall (September, October, November) and Winter (December, January, February), for the same 10 representative models as in Table 2. The values correspond to all 12 datasets and the full out-of-sample period. A heat map is used to indicate better (→ green) and worse (→ red) performing models.

<table>
<thead>
<tr>
<th>Season</th>
<th>mean_HoW</th>
<th>naive</th>
<th>expert_HoW,_al</th>
<th>24_AR_HoW</th>
<th>24_VAR_HoW</th>
<th>24_lasso_HoW,_al</th>
<th>24_lasso_HQC,_HoW,_al</th>
<th>AR_HoW</th>
<th>24_lasso_HQC,_HoW,_p</th>
<th>AR_lasso_HQC,_HoW,_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>121.21</td>
<td>44.35</td>
<td>5.34</td>
<td>12.09</td>
<td>8.12</td>
<td>1.89</td>
<td>1.42</td>
<td>3.93</td>
<td>2.39</td>
<td>1.90</td>
</tr>
<tr>
<td>Summer</td>
<td>144.52</td>
<td>48.77</td>
<td>3.67</td>
<td>11.09</td>
<td>7.94</td>
<td>0.82</td>
<td>1.26</td>
<td>2.80</td>
<td>1.57</td>
<td>2.11</td>
</tr>
<tr>
<td>Fall</td>
<td>74.78</td>
<td>49.40</td>
<td>3.38</td>
<td>10.41</td>
<td>5.73</td>
<td>1.49</td>
<td>1.38</td>
<td>0.78</td>
<td>1.04</td>
<td>0.80</td>
</tr>
<tr>
<td>Winter</td>
<td>92.08</td>
<td>50.65</td>
<td>3.52</td>
<td>15.19</td>
<td>4.23</td>
<td>1.21</td>
<td>1.21</td>
<td>2.69</td>
<td>2.32</td>
<td>1.86</td>
</tr>
</tbody>
</table>

univariate lasso^{HQC}_{HoW,p} is right next to it (being the best for EPEX.CH and NP.DK2). In general, looking at the aggregate m.p.d.f.b\_MAE values in Table 3 we can observe that the annual ranking of the models (see Table 2) is preserved in the Spring, Summer and Winter. However, in the Fall the univariate models have an edge over the multivariate lasso models. A plausible explanation may be that in the majority of analyzed markets the electricity prices and their volatility tend to increase towards the end of the calendar year. The univariate models, by taking into account all hourly prices in the past week, seem to be able to adapt quicker to the increasing prices.

5.2. Testing for significant differences in the forecasting performance

The MAE values analyzed in Section 5.1 can be used to provide a ranking of models. However, the power to draw statistically significant conclusions on the outperformance of the forecasts of one model by those of another is limited – even given their standard errors – since the dependency structure between the errors (or the MAE’s) is neglected. Therefore, we also computed the Diebold and Mariano (1995) test (abbreviated DM) which takes the correlation structure into account. It tests forecasts of each pair of models against each other.

5.2.1. The ‘multivariate’ DM test

In the EPF literature, the DM test is usually performed separately for each of the 24 hours of the day (see Weron, 2014). We also do this here. But first let us introduce a different approach, where only one statistic for each pair of models is computed based on the 24-dimensional vector of errors for each day; we call the resulting DM test multivariate or vectorized. Therefore, denote by $\hat{\varepsilon}_{X,d} = [\hat{\varepsilon}_{X,d,1}, \ldots, \hat{\varepsilon}_{X,d,24}]'$ and $\hat{\varepsilon}_{Y,d} = [\hat{\varepsilon}_{Y,d,1}, \ldots, \hat{\varepsilon}_{Y,d,24}]'$ the vectors of out-of-sample errors for day $d$ of models $X$ and $Y$, respectively. Then the multivariate loss differential series:

$$\Delta_{X,Y,d} = ||\hat{\varepsilon}_{X,d}||_1 - ||\hat{\varepsilon}_{Y,d}||_1,$$

(22)

defines the differences of errors in the $|| \cdot ||_1$-norm, i.e., $||\hat{\varepsilon}_{X,d}||_1 = \sum_{i=1}^{24} |\hat{\varepsilon}_{X,d,i}|$. For each model pair and each dataset we compute the $p$-value of two one-sided DM tests: (i) a test with the null hypothesis $H_0 : E(\Delta_{X,Y,d}) \leq 0$, i.e., the outperformance of the forecasts of model $Y$ by those
of model $X$, and (ii) the complementary test with the reverse null $H_R^0 : E(\Delta_{X,Y,d}) \geq 0$, i.e., the outperformance of the forecasts of model $X$ by those of model $Y$. As in the standard DM test, we assume that the loss differential series is covariance stationary.

In Figure 3 we summarize the results for the 10 considered models, for each market. We use a heat map to indicate the range of the $p$-values – the closer they are to zero (→ dark green) the more significant is the difference between the forecasts of a model on the X-axis (better) and the forecasts of a model on the Y-axis (worse). In other words, if in a given row each square is green then the forecasts of models on the X-axis are significantly better then those of the model on the Y-axis. For instance, for all markets the first row is always dark green indicating that the forecasts of $\text{mean}_{HoW}$ are significantly outperformed by those of all other models. Likewise, for all markets the second row is dark green except for the first column indicating that the forecasts of the naive model are always significantly outperformed by those of all other models but $\text{mean}_{HoW}$.

Note, that the $p$-values of the DM test are symmetric around 0.5, so if the standard test with null $H_0$ yields a $p$-value of 0.05, then the complementary test with the reverse null, i.e., $H_R^0$, returns a $p$-value of 0.95. However, the charts in Figure 3 break the symmetry around the diagonal since the scale is capped at 0.1 (i.e., 10%; for better exposition of the relevant results). Thus if the standard test returns a $p$-value of, say, 0.85, then the complementary test yields a $p$-value of 0.15, and both are indicated by black squares. So neither the forecasts of the model on the X-axis are significantly better than those of the model on the Y-axis nor vice versa, as is the case, e.g., for BELPEX.BE and models $\text{24AR}_{HoW}$ and $\text{VAR}_{HoW}$.

Looking closely at Figure 3 we can observe that for eight datasets (BELPEX.BE, EPEX.CH, EPEX.FR, EXAA.DE+AT, GEFCom2014, NP.SYS, OMIE.ES and OMIE.PT) the forecasts of the best according to MAE multivariate lasso model, i.e., $\text{24lasso}_{HQ\text{C}}^{DoW,p,nl}$, are not significantly worse than those of the best univariate lasso model, i.e., $\text{lasso}_{HoW,p}^{HQ\text{C}}$. On the other hand, the forecasts of the latter are for eight markets (BELPEX.BE, EPEX.DE+AT, EPEX.FR, EXAA.DE+AT, GEFCom2014, NP.DK1, NP.DK2 and OTE.CZ) not significantly worse than the forecasts of $\text{24lasso}_{HQ\text{C}}^{DoW,p,nl}$. Thus, there are four markets (BELPEX.BE, EPEX.FR, EXAA.DE+AT and GEFCom2014) where we cannot decide which model is better based on the considered DM test.

5.2.2. The standard DM test and the performance across the hours

To get a better understanding of how the significance changes across the hours we now consider the standard DM test, for which the loss differential series is given by (see also Bordignon et al., 2013; Nowotarski et al., 2014; Ziel et al., 2015b; Nowotarski and Weron, 2016; Uniejewski et al., 2016):

$$\Delta_{X,Y,d,h} = |\hat{\epsilon}_{X,d,h}| - |\hat{\epsilon}_{Y,d,h}|.$$  \hspace{1cm} (23)

We perform two one-sided DM tests at the 5% significance level: (i) a test with the null hypothesis $H_0 : E(\Delta_{X,Y,d,h}) \leq 0$, i.e. the outperformance of the forecasts of model $Y$ by those of model $X$, and (ii) the complementary test with the reverse null $H_R^0 : E(\Delta_{X,Y,d,h}) \geq 0$, i.e. the outperformance of the forecasts of model $X$ by those of model $Y$. Note, that like in the above cited studies, we assume that forecasts for consecutive days, hence loss differentials are covariance stationary. Note also, that compared to the multivariate DM test approach discussed above, we now perform the test at one significance level (5%) and focus on binary variables representing the passing or not of the
Figure 3: Results of the ‘multivariate’ (or ‘vectorized’) DM test defined by the multivariate loss differential series in Eqn. (22). We use a heat map to indicate the range of the p-values – the closer they are to zero (→ dark green) the more significant is the difference between the forecasts of a model on the X-axis (better) and the forecasts of a model on the Y-axis (worse).
test for a particular hour, not the \( p \)-values themselves.

In Figure [4] we summarize the DM results for all datasets. Namely, we sum the number of significant differences in forecasting performance across the 24 hours and use a heat map to indicate the number of hours for which the forecasts of a model on the X-axis are significantly better than those of a model on the Y-axis. If the forecasts of a model on the X-axis are significantly better for all 24 hours of the day, we indicate this by a white square. On the other hand, if the forecasts of a model on the X-axis are not significantly better for any hour, we plot a black square. Naturally, the diagonal (gray crosses on black squares) should be ignored as it concerns the same model on both axes. Columns with many non-black squares (the more green or white the better) indicate that the forecasts of a model on the X-axis are significantly better than the forecasts of many of its competitors. Conversely, rows with many non-black squares mean that the forecasts of a model on the Y-axis are significantly worse than the forecasts of many of its competitors. For instance, for the EPEX.CH dataset, the white row for the mean\(_{HoW}\) benchmark indicates that the forecasts of this simple model are significantly worse than the forecasts of all of its competitors for all 24 hours, while the black column for mean\(_{HoW}\) means that not a single competitor produces significantly worse forecasts than this benchmark, even for a single hour of the day.

For the more sophisticated lasso models we cannot draw such clear cut conclusions. There is no model that would beat the competitors for all 24 hours of the day. Moreover, for most markets the forecasts of the best univariate and multivariate models are significantly better than those of other structures only for some hours of the day. This shows that the statements about the significance of results we were able to draw from the multivariate DM tests presented in Figure [5] are mainly due to a significantly better performance for a few hours of the day. More interestingly, we see that for many markets, there is at least one hour (indicated by a square that is at least red) where both the forecasts of 24lasso\(_{HoW,p,nl}\) are significantly better than those of lasso\(_{HoW,p}\) and vice versa. For example, for the BELPEX.BE dataset the univariate lasso model shows significantly better forecasts for seven hours of the day and significantly worse for five hours.

In an attempt to better understand the performance of the models across the hours of the day, in Figure [5] we plot the \( p \)-values of the 24 hourly DM tests for the best multivariate vs. the best univariate lasso models. The Y-axis is capped at 0.5, so we either plot a red circle (→ 24lasso\(_{HoW,p,nl}\) yields better forecasts) or a blue square (→ lasso\(_{HoW,p}\) yields better forecasts). Clearly, no universal daily pattern can be observed. If anything, more often the multivariate specification outperforms the univariate in the morning hours (for BELPEX.BE, EPEX.CH, EPEX.FR, OMIE.ES, OMIE.PT and except for hour 1 also for GEFCom2014), whereas the univariate more often outperforms the multivariate in the late evening/night hours (for BELPEX.BE, EPEX.DE+AT, EXAA.DE+AT, two Danish and two Iberian markets). This is in contrast to what Ziel (2016a) concludes for relatively simple models from both model classes – univariate models (like AR\(_{HoW}\)) perform better for the first half of the day, whereas similar structures within a multivariate framework (like 24AR\(_{HoW}\)) are better in the second half of the day. This discrepancy may be due to the complexity (higher number of parameters in this study) or the calibration of the models (different information criteria; introduction of the HQC criterion in this study, which apparently outperforms AIC and BIC, see Appendix C).

Last but not least, the observation that for some hours of the day one model structure domina-
Figure 4: Results of the 24 hourly DM tests at the 5% level, defined by the loss differential series in Eqn. (23). We sum the number of significant differences in forecasting performance across the 24 hours and use a heatmap to indicate the number of hours for which the forecasts of a model on the X-axis are significantly better than those of a model on the Y-axis. A white square indicates that forecasts of a model on the X-axis are better for all 24 hours, while a black square that they are not better for a single hour.
Figure 5: Results of the 24 hourly DM tests, as defined by the loss differential series in Eqn. (23), for the best multivariate vs. the best univariate lasso models. We plot the p-values for the standard test with null $H_0$ (red circles; $\rightarrow 24lasso_{HoW,p}$ yields better forecasts) or the complementary test with reverse null $H_0^*$ (blue squares; $\rightarrow lasso_{HoW,p}^{HQC}$ yields better forecasts), whichever is smaller. The dashed green line represents the 5% significance level.
Table 4: Mean Absolute Errors (MAE) for the better (ex-post) of the two best lasso models, i.e., $\texttt{24lasso}^{\text{HQC}}_{\text{DoW,p,nl}}$ and $\texttt{lasso}^{\text{HQC}}_{\text{HoW,p}}$, and their arithmetic average. Note, that combining yields an improvement for all 12 datasets. According to the multivariate DM-test, defined by Eqn. (22), it is significant at the 5% level for eight markets (denoted by *).

<table>
<thead>
<tr>
<th></th>
<th>BELPEX.BE</th>
<th>EPEX.CH</th>
<th>EPEX.DE+AF</th>
<th>EPEX.FR</th>
<th>EMAA.DE+AF</th>
<th>EXAA.DE+AF</th>
<th>GEFCom2014</th>
<th>NP.DK1</th>
<th>NP.DK2</th>
<th>NP.SYS</th>
<th>OMIE.ES</th>
<th>OMIE.PT</th>
<th>OTE.CZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference in MAE</td>
<td>0.104</td>
<td>0.035</td>
<td>0.041</td>
<td>0.061</td>
<td>0.050</td>
<td>0.087</td>
<td>0.037</td>
<td>0.034</td>
<td>0.006</td>
<td>0.031</td>
<td>0.044</td>
<td>0.038</td>
<td></td>
</tr>
<tr>
<td>Improvement in %</td>
<td>1.75*</td>
<td>0.89*</td>
<td>0.81*</td>
<td>1.27*</td>
<td>1.22*</td>
<td>1.30</td>
<td>0.67*</td>
<td>0.79*</td>
<td>0.33</td>
<td>0.52</td>
<td>0.71</td>
<td>0.85*</td>
<td></td>
</tr>
</tbody>
</table>

Ties another and vice versa gives grounds to believe (or expect) that combining their forecasts will yield further improvements. Although forecast combinations are not the focus of this study, we motivate this conclusion by considering a simple arithmetic average of the best two lasso models: $\texttt{24lasso}^{\text{HQC}}_{\text{DoW,p,nl}}$ and $\texttt{lasso}^{\text{HQC}}_{\text{HoW,p}}$; for recent studies involving forecast averaging we refer to Bordignon et al. (2013), Nowotarski et al. (2014), Weron (2014), Raviv et al. (2015), Gaillard et al. (2016), Marcjasz et al. (2018) and Nowotarski and Weron (2018), among others. In Table 4 we report the MAE values for the better of the two lasso models (this is in row ‘Better of the two’) and their arithmetic average (row ‘Combination’). Clearly, the simple arithmetic average (the weights, i.e., $\frac{1}{2}$ for each model, are obviously chosen ex-ante) gives an improvement in forecasting accuracy (over the ex-post selected lasso model) for all 12 datasets. According to the multivariate DM-test, defined by Eqn. (22), it is significant at the 5% level for the majority of markets. The improvement of 1.30% for the GEFCom2014 dataset is not significant probably due to the much shorter test period.

5.3. Variable selection

In this section we analyze the structures of the best multivariate and univariate lasso models. Like Uniejewski et al. (2016), we count the number of times a given explanatory variable was selected (its coefficient is different from zero) for $\texttt{24lasso}^{\text{HQC}}_{\text{DoW,p,nl}}$ (see Tables 5 and 6) or $\texttt{lasso}^{\text{HQC}}_{\text{HoW,p}}$ (see Table 9). However, unlike Uniejewski et al. (2016), we do not present the numbers themselves, but the percentages (over all days in the out-of-sample test periods and across all 12 datasets). Heat maps are used to indicate more (→ green) and less (→ red) commonly-selected variables. Several interesting conclusions can be drawn.

In particular, for the multivariate lasso model, see Eqn. (13), we observe that:

- For the autoregressive parameters, i.e., $\phi_{h,k,l,0}$, the most important is lag $k = 1$, which refers to electricity prices of the previous day, see Table 5 and compare with Tables 6 and 7. For $k = 1$ we observe that the diagonal elements, $\phi_{h,1,1,0}$, and the last hour of the previous day, $\phi_{h,1,24,0}$, are much more ‘green’ than the remaining variables. However, the diagonal elements seem to be less important during the working hours, whereas $\phi_{h,1,24,0}$ shows high relevance for
all $h$. This impact of hour 24 is likely the reason for the outperformance of the $24$ARHoW model by VARHoW, see Table 2. Even though both models have the same structure in terms of variables used, only VARHoW captures the ‘last hour of the day’ effect across all hours.

- For the less relevant autoregressive parameters at lag $k = 1$, i.e., $\phi_{h,1,0}$, the lower triangle with $l < h$ carries most of the information. This is interesting because this triangle represents all prices $P_{d-1,l}$ which are closer (in time) to the predicted price, i.e., $P_{d,h}$, than the prices 24 hours ago, i.e., $P_{d-1,h}$, represented the by the ‘diagonal’ in Table 5. This relationship seems to carry the most information for the night hours ($l = 1, 2, \ldots, 6$) and the evening hours ($l = 18, 19, \ldots, 24$).

- For the autoregressive parameters with lag $k > 1$, see Tables 5-7, generally only the diagonal elements $\phi_{h,k,0}$ show some importance. Seldom we observe a lag-importance ‘island’ (e.g., for $\phi_{h,2,5,0}$, $\phi_{h,2,23,0}$ or $\phi_{h,7,23,0}$ in the night hours) that could justify the complex model parametrization.

- The non-linear minimum and maximum effects, $\phi_{h,k,l,\text{min}}$ and $\phi_{h,k,l,\text{max}}$, are only relevant for $k = 1$, i.e., the effect of previous day’s minimum/maximum price, see Table 8. In general, it seems that the minimum is more important, especially in the first six hours of the day. In contrast, the maximum seems to be more important for the late morning hours ($h = 8, 9, 10$). To some extent, these temporal differences are also visible in Tables 2 and 3 in Uniejewski et al. (2016). Interestingly, they conclude that the maximum ‘is slightly more influential’ than the minimum, however, both in our and their study the differences are rather small.

- The day-of-the-week dummies, i.e., $\phi_{h,0,0,j}$, are in general very important, especially the Monday, Saturday and Sunday ($j = 1, 6, 7$) dummies, see Table 8. This means that the commonly used design of expert models is appropriate (see e.g. Misiorek et al. 2006; Weron and Misiorek 2008; Serinaldi 2011; Kristiansen 2012; Nowotarski et al. 2014; Gaillard et al. 2016; Maciejowska et al. 2016; Nowotarski and Weron 2018; Uniejewski et al. 2016; Ziel 2016a). The Tuesday and Friday ($j = 2, 5$) dummies are less important, with the latter one only for the evening hours.

- The periodic parameters, $\phi_{h,1,j}$ and $\phi_{h,1,24,j}$, exhibit relevance, see Table 8. However, in contrast to the day-of-the-week dummies ($\phi_{h,0,0,j}$), $\phi_{h,1,j}$’s are more important during the working days, especially Tuesday, Wednesday and Friday ($j = 3, 4, 5$).

For the univariate lasso model, see Eqn. (19), we observe that:

- The autoregressive parameters which model the dependency on the previous hour, i.e., $\phi_{1,h}$, have a clear pattern. Lags around multiples of 24 are very important, e.g. 23, 24, 25 and 26 or 47, 48, 49 and 50. However, the remaining lags exhibit moderate importance and only for the first 24 hours, see Table 9.

- For the intercepts and periodic parameters, i.e., $\phi_{0,k}$, $\phi_{2,k}$ and $\phi_{3,k}$, the patterns are not that obvious. However, almost every parameter seems to have relevance at least for some markets (as there are almost no red backgrounds). We also see that lags being a multiple of 24 tend to have more importance than the other parameters, which is most clearly visible for the intercepts: $\phi_{0,24}, \phi_{0,48}$, etc.

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• Another interesting observation is that there is another group of lag-importance ‘islands’. These occur at lags of order 7 and 8, e.g. \(24+7=31\) and \(24+8=32\) or \(48+7=55\) and \(48+8=56\).

Finally, we should note that for the multivariate model the non-linear effects exhibit moderate importance and increase the overall model fit. However, for the univariate model, the non-linear effects do not improve the model performance. The reason may be that we only use two parameters to capture the minimum/maximum effects, even though the effect seems to be periodic. An introduction of non-linear periodic effects in the univariate setting may lead to a further improvement of the predictive accuracy.

6. Conclusions and guidelines for energy forecasters

We have conducted an extensive empirical study on short-term electricity price forecasting (EPF) to address the long-standing question if the optimal model structure for EPF is univariate or multivariate. We provide evidence that despite a minor edge in predictive performance overall, as measured by the linear (MAE) and quadratic (RMSE) error measures and the mean percentage deviation from the best performing model (m.p.d.f.b.), the multivariate modeling approach does not uniformly outperform the univariate one across all datasets, seasons of the year or hours of the day, and at times is outperformed by the latter. These fluctuations in forecasting performance across the hours of the day can be utilized, however, via model averaging or combining forecasts. As illustrated in the paper, a simple arithmetic average of the forecasts of the best multivariate and the best univariate lasso model beats the better (ex-post) of the two for all 12 considered datasets.

When analyzing model performance in the four seasons of the year, we find that the annual ranking of the models is preserved in the Spring, Summer and Winter. However, in the Fall the univariate models have an edge over the multivariate lasso models. A plausible explanation may be that in the majority of analyzed markets the electricity prices and their volatility tend to increase towards the end of the calendar year. The univariate models, by taking into account all hourly prices in the past week, seem to be able to adapt quicker to the increasing prices. Interestingly, team TOLOLO used a similar approach to win the Price Track of the GEFCom2014 competition (see Gaillard et al., 2016). For three tasks corresponding to days in December (i.e., late Fall/early Winter) they used a specifically designed model, different from the models for the remaining tasks (corresponding to Summer months). These results suggest that the predictive efficiency may be further increased by designing different models for different seasons of the year.

Also regarding performance across the markets, there is some variability in forecasting accuracy, which may be a result of differences in the generation mix or market regulations. The latter may be accounted for when constructing fundamental models (like the X-model of Ziel and Steinert, 2016 that focuses on modeling the bidding behavior), however, we do not see a straightforward way of incorporating such information into statistical models. Yet, it is always one of the multivariate lasso models estimated using the HQC criterion or the univariate lasso model with periodic effects, also estimated using the HQC criterion, that yields the best performance. Given that the three multivariate lasso models never perform badly, they are recommended for EPF in general.

Concerning variable (or feature) selection, analyzing the structures of the best multivariate and univariate lasso models, we have identified the most important variables and thus provided
Table 5: Mean occurrence (in %) of the multivariate lasso model parameters across all 12 datasets and the full out-of-sample test period. Columns represent the hours and rows the parameters of the 24lassoHQC model, see Eqs. (13) for details. A heat map is used to indicate more (→ green) and less (→ red) commonly-selected variables. Continued in Table 6.
Table 6: Mean occurrence (in %) of the multivariate lasso model parameters across all 12 datasets and the full out-of-sample test period. Columns represent the year and rows the parameters of the Q24lasso model, see Eqn. (13) for details. A heat map is used to indicate more (→ green) and less (→ red) commonly-selected variables. Continued in Table 7.

| Year | \( \phi_1 \) | \( \phi_2 \) | \( \phi_3 \) | \( \phi_4 \) | \( \phi_5 \) | \( \phi_6 \) | \( \phi_7 \) | \( \phi_8 \) | \( \phi_9 \) | \( \phi_{10} \) | \( \phi_{11} \) | \( \phi_{12} \) | \( \phi_{13} \) | \( \phi_{14} \) | \( \phi_{15} \) | \( \phi_{16} \) | \( \phi_{17} \) | \( \phi_{18} \) | \( \phi_{19} \) | \( \phi_{20} \) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 2006 | 0.38    | 1.40    | 0.30    | 0.52    | 1.31    | 0.30    | 0.49    | 0.28    | 0.34    | 0.36    | 0.29    | 0.36    | 0.34    | 0.34    | 0.30    | 0.30    | 0.30    | 0.30    | 0.30    |
| 2007 | 0.40    | 1.39    | 0.31    | 0.53    | 1.31    | 0.30    | 0.50    | 0.28    | 0.34    | 0.37    | 0.30    | 0.37    | 0.35    | 0.35    | 0.31    | 0.31    | 0.31    | 0.31    | 0.31    |
| 2008 | 0.41    | 1.38    | 0.32    | 0.55    | 1.32    | 0.30    | 0.51    | 0.29    | 0.35    | 0.38    | 0.31    | 0.38    | 0.36    | 0.36    | 0.32    | 0.32    | 0.32    | 0.32    | 0.32    |
| 2009 | 0.42    | 1.37    | 0.33    | 0.57    | 1.33    | 0.30    | 0.52    | 0.30    | 0.36    | 0.40    | 0.32    | 0.39    | 0.38    | 0.38    | 0.34    | 0.34    | 0.34    | 0.34    | 0.34    |
| 2010 | 0.43    | 1.36    | 0.34    | 0.59    | 1.34    | 0.31    | 0.54    | 0.31    | 0.37    | 0.42    | 0.33    | 0.40    | 0.39    | 0.39    | 0.35    | 0.35    | 0.35    | 0.35    | 0.35    |
| 2011 | 0.44    | 1.35    | 0.35    | 0.61    | 1.35    | 0.32    | 0.55    | 0.32    | 0.38    | 0.44    | 0.34    | 0.42    | 0.41    | 0.41    | 0.37    | 0.37    | 0.37    | 0.37    | 0.37    |
| 2012 | 0.45    | 1.34    | 0.36    | 0.63    | 1.36    | 0.33    | 0.56    | 0.33    | 0.39    | 0.46    | 0.35    | 0.44    | 0.43    | 0.43    | 0.39    | 0.39    | 0.39    | 0.39    | 0.39    |
| 2013 | 0.46    | 1.33    | 0.37    | 0.65    | 1.37    | 0.34    | 0.57    | 0.35    | 0.40    | 0.48    | 0.36    | 0.46    | 0.45    | 0.45    | 0.41    | 0.41    | 0.41    | 0.41    | 0.41    |
| 2014 | 0.47    | 1.32    | 0.38    | 0.67    | 1.38    | 0.35    | 0.58    | 0.36    | 0.42    | 0.50    | 0.37    | 0.49    | 0.48    | 0.48    | 0.44    | 0.44    | 0.44    | 0.44    | 0.44    |
| 2015 | 0.48    | 1.31    | 0.39    | 0.69    | 1.39    | 0.36    | 0.59    | 0.37    | 0.43    | 0.52    | 0.38    | 0.51    | 0.50    | 0.50    | 0.46    | 0.46    | 0.46    | 0.46    | 0.46    |
| 2016 | 0.49    | 1.30    | 0.40    | 0.71    | 1.40    | 0.37    | 0.60    | 0.38    | 0.45    | 0.54    | 0.39    | 0.54    | 0.53    | 0.53    | 0.49    | 0.49    | 0.49    | 0.49    | 0.49    |
| 2017 | 0.50    | 1.29    | 0.41    | 0.73    | 1.41    | 0.38    | 0.61    | 0.39    | 0.47    | 0.56    | 0.40    | 0.57    | 0.56    | 0.56    | 0.52    | 0.52    | 0.52    | 0.52    | 0.52    |
| 2018 | 0.51    | 1.28    | 0.42    | 0.75    | 1.42    | 0.39    | 0.62    | 0.40    | 0.49    | 0.58    | 0.41    | 0.59    | 0.59    | 0.59    | 0.55    | 0.55    | 0.55    | 0.55    | 0.55    |
| 2019 | 0.52    | 1.27    | 0.43    | 0.77    | 1.43    | 0.40    | 0.63    | 0.41    | 0.51    | 0.60    | 0.42    | 0.61    | 0.61    | 0.61    | 0.57    | 0.57    | 0.57    | 0.57    | 0.57    |
| 2020 | 0.53    | 1.26    | 0.44    | 0.79    | 1.44    | 0.41    | 0.64    | 0.42    | 0.53    | 0.62    | 0.43    | 0.63    | 0.63    | 0.63    | 0.59    | 0.59    | 0.59    | 0.59    | 0.59    |

For the full year, see Table 7.
Table 7: Mean occurrence (in %) of the multivariate lasso model parameters across all 12 datasets and the full out-of-sample test periods. Columns represent the hours and rows the parameters of the \texttt{24lassoDouglas} model, see Eqn. \((13)\) for details. A heat map is used to indicate more (→ green) and less (→ red) commonly-selected variables. Continued in Table \([\text{S}]\).
Table 8: Mean occurrence (in %) of the multivariate lasso model parameters across all 12 datasets and the full out-of-sample test period. Columns represent the hours and rows the parameters of the $24\text{lasso}^{HQC}_{\text{DoW,p,nl}}$ model, see Eqn. (13) for details. A heat map is used to indicate more (→ green) and less (→ red) commonly-selected variables.

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Table 9: Mean occurrence (in %) of the univariate lasso model parameters across all 12 datasets and the full out-of-sample test period. The columns represent the intercept ($\phi_{0,k}$), the autoregressive terms ($\phi_{1,k}$), the periodic effects of lag 1 ($\phi_{2,k}$), and the periodic effects of lag 2 ($\phi_{3,k}$) of the lasso model, see Eqn. [17]. For details, note that this model does not include the non-linear effects, i.e., $\phi_{1,k} = \phi_{2,k} = 0$. A heat map is used to indicate more ($\phi$) and less ($\phi$) commonly-selected variables.
guidelines to structuring better performing expert models. In particular, we have confirmed the high explanatory power of last day’s prices for the same or neighboring hours, of last day’s prices for midnight and of the price for the same hour a week earlier. However, more importantly, we have found the periodic effects (daily dummies multiplied by the last day’s prices for the same hour or for midnight) to play a very important role. Hence, like Uniejewski et al. (2016), we strongly suggest to incorporate periodic structures not only in expert models, but also in general model designs.

Finally, we should comment on converting univariate models into multivariate and vice versa. This possibility can give ideas for modeling approaches in the ‘other modeling world’. For instance, if we observe a clear effect for a particular parameter in the multivariate setting, then the corresponding effect should be important for univariate modeling as well. However, when re-writing multivariate models into univariate form and vice versa, the error structures change. So implicitly, in a multivariate specification the residual variance is assumed to be different for the 24 models, whereas for the univariate models the considered estimation methods assume that the variance is the same across all hours. Here, iterative reweighting schemes can help to incorporate variance changing effects, especially for univariate approaches (see e.g., Ziel et al., 2015a; Ziel, 2016b).

Appendix A. The set of models

In this Appendix we define the remaining models from classes C3-C8 that were considered in the empirical study, but are not discussed and compared in Sections 4 and 5. They are briefly evaluated in terms of WMAE and m.p.d.f.b. in Appendix C below.

Appendix A.1. Expert models (class C3)

For the asinh-transformed price on day $d$ and hour $h$, the generic class of expert models used in this paper is defined by:

$$Y_{d,h} = \beta_{h,1} + \beta_{h,2}Y_{d-1,h} + \beta_{h,3}Y_{d-2,h} + \beta_{h,4}Y_{d-7,h} + \beta_{h,5}Y_{d-1,min} + \beta_{h,6}Y_{d-1,max} + \beta_{h,7}Y_{d-1,24}$$

$$+ \sum_{j=1}^{7} \beta_{h,7+j}DoW_{d,h}^j + \sum_{j=1}^{7} \beta_{h,14+j}DoW_{d,h}^j Y_{d-1,h} + \sum_{j=1}^{7} \beta_{h,21+j}DoW_{d,h}^j Y_{d-1,24} + \epsilon_{d,h}. \quad (A.1)$$

Note, that due to collinearity $\beta_{h,14}$, $\beta_{h,21}$ and $\beta_{h,28}$ can be dropped, so the model has effectively 25 parameters (for $h = 24$ only 18). We estimate the parameters using OLS.

Such a model is denoted by $\text{expert}_{DoW,p,nl}$. Compared to the $\text{expert}_{DoW,nl}$ model considered in Sections 4 and 5, the above formula additionally includes periodic effects (hence subscript $p$ in the model name). The second sum in Eqn. (A.1) is a consequence of the experience gained by TEAM POLAND during the GEFCom2014 competition; one of the conclusions of Maciejowska and Nowotarski (2016) was that it could be beneficial to use different model structures for different days of the week, not only different parameter sets. The third sum in Eqn. (A.1) is an innovation.
introduced in this study to allow for different model structures including the previous day’s price at midnight, i.e., \( Y_{d-1,24} \), for different days of the week. We consider three special cases of the expert_{DoW,p,nl} model:

2. expert_{DoW,nl} without periodic effects (used in Sections 4 and 5),
3. expert_{DoW,p} without non-linear effects,
4. expert_{DoW} without periodic and non-linear effects.

Furthermore, if we restrict the three sums in the second row of Eqn. (A.1) to sum only over Monday, Saturday and Sunday, i.e., \( j = 1, 6, 7 \), then we receive the next four experts:

5. expert_{p,nl} – the full model,
6. expert_{nl} without periodic effects,
7. expert_{p} without non-linear effects,
8. expert without periodic and non-linear effects.

There is an important difference between models defined in Eqn. (A.1) and the expert models considered by Uniejewski et al. (2016). In the latter article no intercept is included in the formulas, i.e., \( \beta_{h,1} = 0 \). Instead \( Y_{d,h} \) is de-meaned by the daily mean of hourly frequency, i.e., \( \bar{Y}_{HoD,d,h} \) in our notation of Section 4.1. Replacing \( Y_{d,h} \) by \( (Y_{d,h} - \bar{Y}_{HoD,d,h}) \) and removing the intercept \( \beta_{h,1} \) in Eqn. (A.1) yields the corresponding models. We denote them by superscript *:

9-10. expert^{*}_{DoW,p,nl} and expert^{*}_{p,nl} – the full models,
11-12. expert^{*}_{DoW,nl} and expert^{*}_{nl} without periodic effects,
13-14. expert^{*}_{DoW,p} and expert^{*}_{p} without non-linear effects,
15-16. expert^{*}_{DoW} and expert^{*} without periodic and non-linear effects.

Appendix A.2. The second 24AR-type model (class C4)

As an alternative to the 24AR_{HoW} model defined in Section 4.3.2, we consider a specification in which the asinh-transformed price is demeaned with respect to \( \bar{Y}_{HoD,d,h} \), not \( \bar{Y}_{HoW,d,h} \), and modeled as an AR(\( p_{h} \)) process independently for each hour \( h \). The resulting 24AR_{HoD} model is given by:

\[
Y_{d,h} = \bar{Y}_{HoD,d,h} + \phi_{0,h} + \sum_{k=1}^{p_{h}} \phi_{k,h}(Y_{d-k,h} - \bar{Y}_{HoD,d,h}) + \epsilon_{d,h}, \tag{A.2}
\]

and estimated analogously to 24AR_{HoW}.

Appendix A.3. The second VAR-type model (class C5)

As an alternative to the VAR_{HoW} model defined in Section 4.3.3, we consider the VAR_{HoD} model:

\[
Y_{d} = \bar{Y}_{HoD,d} + \phi_{0} + \sum_{k=1}^{p} \Phi_{k}(Y_{d-k} - \bar{Y}_{HoD,d}) + \epsilon_{d}, \tag{A.3}
\]

where \( Y_{d} = [Y_{d,1}, \ldots, Y_{d,24}]' \) with its mean vector \( \bar{Y}_{HoD,d} \) across all available days in the calibration sample (which corresponds to the 24 possible values of \( \bar{Y}_{HoD,d,h} \)). Analogously to VAR_{HoW}, we calibrate the model by solving the multivariate Yule-Walker equations with \( p_{max} = 8 \).
Appendix A.4. Multivariate lasso models (class C6)

Based on Eqn. (13) and using certain restrictions, we can define the full set of 16 models:

1-4. \text{24lasso}^{IC}_{DoW,p,nl} - the full model,
5-8. \text{24lasso}^{IC}_{DoW,nl} without the periodic parameter terms (i.e., with $\phi_{h,1,h,j} = \phi_{h,1,24,j} = 0$),
9-12. \text{24lasso}^{IC}_{DoW,p} without the non-linear effects (i.e., with $\phi_{h,k,min,0} = \phi_{h,k,max,0} = 0$),
13-16. \text{24lasso}^{IC}_{DoW} without the non-linear and periodic effects,

where the superscript IC (= AIC, HQC, BIC, OLS) denotes the information criterion used.

Because of the nature of the shrinkage factor in Eqn. (17), making $\lambda$ sufficiently large will cause some of the coefficients to be exactly zero. Obviously, selecting a good value of $\lambda$ for the lasso is critical. We choose it such that an in-sample information criterion is maximized. To this end, we consider the generalized information criterion \( GIC_k(\beta) \) (see Stoica and Selen, 2004):

\[
GIC_k(\beta) = RSS(\beta) + \kappa K \beta \sigma^2,
\]  
(A.4)

where $\kappa$ is an information criterion type parameter, RSS(\( \beta \)) is the residual sum of squares associated with $\beta$, $K \beta$ are the non-zero parameters in $\beta$ and $\sigma^2$ is the variance of the residuals that we estimate by the sample variance. We consider the following special cases:

- the Akaike Information Criterion (AIC) with $\kappa = 2$, as used in the EPF context by Ziel et al. (2015a),
- the Hannan-Quinn Information Criterion (HQC) with $\kappa = 2 \log(\log(n))$,
- and the Bayesian Information Criterion (BIC; also known as the Schwarz criterion) with $\kappa = \log(n)$, as used in the EPF context by Ziel (2016a),

where $n$ is the sample size and $p$ is the number of possible parameters in the model (in fact, both are the dimensions of the model parameters of the regression matrix). Next to the lasso estimation approach, we also consider the OLS solution, with IC=OLS. It can be interpreted as an information criterion based solution with $\kappa = 0$. Note, that two HQC-based models, i.e., \text{24lasso}^{HQC}_{DoW,p,nl} and \text{24lasso}^{HQC}_{DoW,nl}, have been analyzed in the empirical study in Section 5.

Appendix A.5. Three other univariate AR models (class C7)

The AR\(HoW\) model is complemented by three univariate AR structures:

\[
Y_t = \bar{Y} + \phi_0 + \sum_{k=1}^{p} \phi_k(Y_{t-k} - \bar{Y}) + \varepsilon_t,
\]  
(A.5)

\[
Y_t = \bar{Y}_{DoW,t} + \phi_0 + \sum_{k=1}^{p} \phi_k(Y_{t-k} - \bar{Y}_{DoW,t}) + \varepsilon_t,
\]  
(A.6)

\[
Y_t = \bar{Y}_{HoD,t} + \phi_0 + \sum_{k=1}^{p} \phi_k(Y_{t-k} - \bar{Y}_{HoD,t}) + \varepsilon_t,
\]  
(A.7)

denoted by AR, AR\(DoW\) and AR\(HoD\), respectively. Like the AR\(HoW\) model, we estimate these three variants by solving the Yule-Walker equations and minimizing the AIC with a maximum order of $p_{\text{max}} = 196$.  

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**Appendix A.6. Univariate lasso models (class C8)**

Given the following general formula:

\[
Y_t = \sum_{k=1}^{168} \phi_{0,k} \text{HoW}_t^k + \sum_{k=1}^{196} \phi_{1,k} Y_{t-k} + \sum_{k=1}^{168} \phi_{2,k} \text{HoW}_t^k Y_{t-1} + \sum_{k=1}^{168} \phi_{3,k} \text{HoW}_t^k Y_{t-24} + \phi_{4,1} Y_{t-24,\text{min}} + \phi_{4,2} Y_{t-24,\text{max}} + \epsilon_t, \tag{A.8}
\]

we can define the full set of 16 models:

1-4. \text{lasso}_{IC}^{\text{HoWp,nl}} - the full model,

5-8. \text{lasso}_{IC}^{\text{HoWp}} without non-linear effects (i.e., with \(\phi_{4,1} = \phi_{4,2} = 0\)),

9-12. \text{lasso}_{IC}^{\text{HoW,nl}} without periodic effects (i.e., with \(\phi_{2,k} = \phi_{3,k} = 0\)),

13-16. \text{lasso}_{IC}^{\text{HoW}} without non-linear and periodic effects,

where the superscript \(\text{IC} = \text{AIC, HQC, BIC, OLS}\) denotes the information criterion used; for estimation details see Section 4.3.4 and Appendix A.4. Note, that two HQC-based models, i.e., \text{24lasso}_{\text{DoWp}}^{\text{HQC}} and \text{24lasso}_{\text{DoW}}^{\text{HQC}} have been analyzed in the empirical study in Section 5.

**Appendix B. Alternative representations**

In this Appendix we provide alternative representations for some of the models considered in the study. This may lead to a better understanding of the autoregressive structures and the relationships between similar the multivariate and univariate modeling frameworks.

For instance, the expert models defined in Section 4.3.1 and Appendix A.1 can be written in a univariate way as well. However, the representation is quite complex, therefore we show the representation only for the \text{expert}_{\text{HoWp,nl}} model defined by Eqn. (A.1). The model is a sparse 168-periodic AR model with non-linear impact:

\[
Y_t = \phi_{0,t} + \sum_{k=1}^{168} \phi_{k,t} Y_{t-k} + \phi_{\text{min},t} Y_{\text{min},t} + \phi_{\text{max},t} Y_{\text{max},t} + \epsilon_t, \tag{B.1}
\]

\[
\phi_{k,t} = \begin{cases} 
\beta_{h,1} + \sum_{j=1}^{7} \beta_{h,7+j} \text{DoW}_t^j, & \text{for } k = 0 \\
\beta_{h,7} \text{HoD}_t^j + \sum_{j=1}^{7} \beta_{h,21+j} \text{HoD}_t^j \text{DoW}_t^j, & \text{for } k = 1, 2, \ldots, 23 \\
\beta_{h,2} + \beta_{h,7} \text{HoD}_t^j + \sum_{j=1}^{7} (\beta_{h,14+j} + \beta_{h,21+j} \text{HoD}_t^j) \text{DoW}_t^j, & \text{for } k = 24 \\
\beta_{h,3}, & \text{for } k = 48 \\
\beta_{h,4}, & \text{for } k = 168 \\
0, & \text{otherwise,}
\end{cases}
\]

\(\phi_{\text{min},t} = \beta_{h,5}, \ \phi_{\text{max},t} = \beta_{h,6}, \ Y_{\text{min},t} = \min(Y_{t-1}), \ Y_{\text{max},t} = \max(Y_{t-1})\) with \(h = \text{mod}(t - 1, 24) + 1, \ d = (t - h)/24, \) and mod as modulo operator with respect to the second argument (here 24), e.g., for \(t = 26\) we receive \(\text{mod}(26 - 1, 24) + 1 = 2.\)
On the other hand, the $24\text{AR}_{\text{HoD}}$ model defined by Eqn. (A.2) and the $\text{VAR}_{\text{HoD}}$ model defined by Eqn. (A.3) can be easily rewritten as univariate models. The former as a sparse 24-periodic AR model:

$$
Y_t = \phi_{0,t} + \sum_{k=1}^{p} \phi_{24k,t} Y_{t-24k} + \epsilon_t \quad \text{with} \quad \phi_{24k,t} = \phi_{24k,h},
$$

where $p = \max_i(p_i)$ and $h = \text{mod}(t - 1, 24) + 1$, and the latter as a 24-periodic AR model:

$$
Y_t = \phi_{0,t} + \sum_{k=1}^{K} \phi_{k,t} Y_{t-k} + \epsilon_t \quad \text{with} \quad \phi_{0,t} = \phi_{0,h},
$$

$$
\phi_{k,t} = \phi_{\text{div}(k+h-1,24),h,\text{mod}(k+h-1,24)+1},
$$

$h = \text{mod}(t - 1, 24) + 1$, $K = 24(p + 1) - 1$, $\phi_{k,i,m}$ as elements of $\Phi_k$ where we set $\phi_{0,i,m} = \phi_{p+i,j,m} = 0$ for all $i > 0$ and div as quotient of the Euclidean division with respect to the second argument. Alternatively, we can represent the $\text{VAR}_{\text{HoD}}$ model as a set of 24 single equations, where for each $h$ we have:

$$
Y_{d,h} = Y_{\text{HoD},d,h} + \phi_{h,0} + \sum_{k=1}^{p} \sum_{j=1}^{24} \phi_{h,j,k}(Y_{d-k,j} - Y_{\text{HoD},d-k,h}) + \epsilon_{d,h}.
$$

Finally, the univariate $\text{AR}$ model, defined by Eqn. (A.5), can be rewritten as a special case of a 24-dimensional VAR process. However, the representation is:

$$
Y_d = \Phi_0^{-1} \phi_0 + \sum_{k=1}^{r} \Phi_0^{-1} \Phi_k (Y_{d-1} - Y_d) + \epsilon_d,
$$

with $\phi_0 = (\bar{Y} + \phi_0) 1$,

$$
\Phi_0 = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 \\
-\phi_1 & 1 & 0 & \cdots & 0 & 0 \\
-\phi_2 & -\phi_1 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
-\phi_{22} & -\phi_{21} & -\phi_{20} & \cdots & 1 & 0 \\
-\phi_{23} & -\phi_{22} & -\phi_{21} & \cdots & -\phi_1 & 1 \\
\end{bmatrix}
$$

and

$$
\Phi_k = \begin{bmatrix}
\phi_{24(k-1)} & \phi_{24(k-1)-1} & \cdots & \phi_{24(k-1)-23} \\
\phi_{24(k-1)+1} & \phi_{24(k-1)} & \cdots & \phi_{24(k-1)-22} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{24(k-1)+23} & \phi_{24(k-1)+22} & \cdots & \phi_{24(k-1)} \\
\end{bmatrix},
$$

and we assume that $\phi_k = 0$ for $k > p$.

Appendix C. Model selection

In Figures C.6 and C.7 we summarize the results for all 58 models. In particular, in Figure C.6 we plot Mean Absolute Errors (MAE) for the full out-of-sample period, as defined by Eqn. (20), for all 58 models and four major markets: EPEX.DE+AT, GEFCOM2014, NP.SYS and OMIE.ES.

From Figure C.7 we can clearly see that the HQC criterion, see Section 4.3.4, leads to the best performing on average lasso models, both multivariate and univariate. Interestingly, only the AIC and BIC criteria have been tried in this context before (see Ziel et al. 2015a; Ziel 2016a).
Figure C.6: Mean Absolute Errors (MAE) for the full out-of-sample period, as defined by Eqn. (20), for all 58 models and four major markets (from top to bottom): EPEX.DE+AT, GECom2014, NR.SYS and OMIE.E.S. The solid and dotted whiskers represent the $2\sigma$ and $3\sigma$ ranges, respectively. The dashed red line indicates the MAE of the best performing model. Gray/white background is used to group models from the same class. For clarity of presentation, in all four panels we use an upper cap, e.g., for the EPEX.DE+AT market all MAE values exceeding 5.5 (EUR/MWh) are set equal to 5.5.
This behavior is, however, not uniform across the markets. For instance, for the NP.DK1 and NP.DK2 datasets lasso\textsubscript{BIC}\textsubscript{DoW} is as good as lasso\textsubscript{HQC}\textsubscript{DoW}. From Figure C.7 we can also see that the HQC-estimated multivariate lasso models on average outperform all univariate lasso models. But the differences are small. On the other hand, when the full model is estimated (i.e., the OLS ‘criterion’), the performance deteriorates but the loss of forecasting accuracy is extreme only for the multivariate lasso models. Probably the 24 times shorter calibration sample of 730 observations is too small for the multi-parameter structure.

The importance of including dummies for all days of the week is clearly visible in Fig. C.7. Every second expert model is better than the preceding model without the DoW component. These results support the observation of Uniejewski et al. (2016) that the weekly seasonality requires better modeling than offered by typically-used expert models. This effect is also visible for the multi-parameter structures – 24AR\textsubscript{HoW} is better than 24AR\textsubscript{HoD}, VAR\textsubscript{HoW} is better than VAR\textsubscript{HoD}, AR\textsubscript{DoW} is better than AR and AR\textsubscript{HoW} is better than AR\textsubscript{HoD}.

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