Practice problems #1: partial derivatives

(1) Find first partial derivatives of f if:

$$f(x,y) = x^3 - 2xy^2 + 4y + 1, \quad f(x,y) = \frac{x}{y^2} - \frac{y}{x^2}, \quad f(x,y) = \sin(xy)\cos(x+y),$$

$$f(x,y) = \arctan\frac{y}{x}, \qquad f(x,y) = x^y, \qquad f(x,y) = \ln(x^2+y^2).$$

(2) Find second partial derivatives of f if:

$$f(x,y) = \frac{x}{\sqrt{x^2+y^2}}, \qquad f(x,y) = \frac{xe^{xy}}{y}.$$

(3) Let $f(x,y) = e^{x/y}\sin(x/y) + e^{y/x}\cos(y/x)$. Show that $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 0$.

(4) Show that the following functions satisfy the Laplace's equation $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$:

$$f(x,y) = \arctan\frac{y}{x}, \qquad f(x,y) = \ln(x^2+y^2).$$

(5) Find the first partial differentials of $z(x,y)$ given implicitly by the equation

$$F(x,y,z(x,y)) = 0,$$

if F is defined by:

$$F(x,y,z) = z^3 - 3x^2y + 6xyz, \qquad F(x,y,z) = x^z + y^z - (x+y)^z.$$

(6) Find the equation of the plane tangent to

$$z = x^2\sqrt{1+2y^2} \qquad \text{at } (x_0,y_0) = (3,2);$$

$$z = \arctan(y/x) \qquad \text{at } (x_0,y_0) = (2,\sqrt{3}).$$

(7) Find all points (x_0,y_0,z_0) such that the plane tangent to $z = 2x^2y + 3y^2$ at (x_0,y_0,z_0) contains $(0,0,0)$.

(8) For functions f given in problem (1), write exact differentials df.

(9) Use differentials to approximately compute the volume and surface area of a cylinder with base of radius 2.003 and height 2.991.

(10) Approximate the maximum possible error and the percent error (or relative error) when z is computed by the formula:

$$z = \frac{\sqrt[3]{x}}{y}, \qquad \text{if } x = 8 \pm 0.125 \text{ and } y = 2 \pm 0.1;$$

$$z = \arctan\frac{y}{x}, \quad \text{if } x = 2,\ y = \sqrt{3}, \text{ both measured with maximal percent error } 1\%.$$