Practice problems #10: Difference equations

(1) Evaluate first few terms of the sequence $x(n)$, knowing that

$$x(n+2)x(n) = \frac{n+2}{n}\left(x(n+1)\right)^2$$

and $x(1) = 1$, $x(2) = 2$. Next, guess the expression for $x(n)$ and show that your guess is indeed a solution.

(2) Does the difference equation

$$x(n+2)x(n) = x(n+1) + 2$$

have a unique solution which satisfies $x(1) = x(2) = 2$? What happens if $x(1) = -3$ and $x(2) = 4$? Hint: in each case evaluate $x(3)$, $x(4)$ and $x(5)$ to see what is happening.

(3) Find $x(n)$, knowing that $x(1) = 0$ and

$$\Delta x(n) = 4n^3 + 4n^2 - 1.$$

(4) Solve homogeneous linear difference equations with constant coefficients:

(a) $x(n+2) - 3x(n+1) + 2x(n) = 0$,

(b) $\Delta^2 x(n) + x(n) = 0$,

(c) $\Delta^2 x(n) - x(n) = 0$,

(d) $\Delta^2 x(n) + 2\Delta x(n) = 0$.

(5) Using Z-transforms, solve Volterra-type difference equations

(a) $\displaystyle x(n+1) = \sum_{k=0}^{n} \frac{x(n-k)}{2^k}$,

(b) $\displaystyle x(n+1) = 1 + \sum_{k=0}^{n} \frac{x(n-k)}{2^k}$,

(c) $\displaystyle x(n+1) = (-1)^n + \sum_{k=0}^{n} \frac{x(n-k)}{2^k}$.

Useful expressions:

$$\mathcal{Z}\left[a^n\right] = \frac{z}{z-a},$$

$$\mathcal{Z}\left[x(n+1)\right] = z\mathcal{Z}\left[x(n)\right] - zx(0),$$

$$\mathcal{Z}\left[x(n) + y(n)\right] = \mathcal{Z}\left[x(n)\right] + \mathcal{Z}\left[y(n)\right],$$

$$\mathcal{Z}\left[\sum_{k=0}^{n} x(n-k)y(k)\right] = \mathcal{Z}\left[x(n)\right]\mathcal{Z}\left[y(n)\right].$$