Practice problems #2: extrema

(1) Find local extrema of:

$$f(x,y) = x^3 + y^3 + 3xy, \qquad f(x,y) = xy(2x+4y+1),$$

$$f(x,y) = x^3 - x^2 + xy^2 + 2xy, \qquad f(x,y) = \frac{2x+y+2}{1+x^2+y^2}.$$

What are the global extrema of these functions?

(2) Find the point on the surface $z = xy - 1$ nearest the origin.

(3) Find positive numbers x, y, z such that $x + y + z = 12$ and xy^2z^3 is a maximum.

(4) Find the equation of the plane through $(1, 1, 2)$ that cuts off the least volume in the first octant.

(5) Let $f(x,y) = (x^2-1)^2 + (x^2y - x - 1)^2$. Show that there is only two critical points of $f(x,y)$, and both are local minima of f.

(6) Observe that there is no fucntion of one real variable with only two critical points, both local minima. Inspect the contour plot of $f(x,y)$ from the previous problem to understand why it is possible for functions of two variables.