## Practice problems #5: line integrals

(1) Find the arc length of (one segment of) the helix

$$\begin{cases} x(t) = r\cos t, \\ y(t) = r\sin t, \\ z(t) = at, \end{cases}$$

where $t \in [0, 2\pi]$.

(2) Find the arc length of (one segment of) the cycloid

$$\begin{cases} x(t) = a(t - \sin t), \\ y(t) = a(1 - \cos t), \end{cases}$$

where $t \in [0, 2\pi]$.

(3) Find the arc length of the cardioid

$$\begin{cases} x(t) = 2(1 - \cos t)\cos t, \\ y(t) = 2(1 - \cos t)\sin t, \end{cases}$$

where $t \in [0, 2\pi]$.

(4) Evaluate $\displaystyle\int_\gamma f(x, y)dl$ if

(a) $f(x, y) = xy^2$, $\gamma$ is a line segment that connects $(-1, -1)$ and $(1, 2)$;
(b) $f(x, y) = xy$, $\gamma$ is a circle of radius 2 with center at $(1, 2)$.

(5) The center of mass of a curve $\gamma$ has coordinates $(x_0, y_0)$, where

$$x_0 = \frac{1}{L}\int_\gamma x\,dl, \qquad y_0 = \frac{1}{L}\int_\gamma y\,dl, \qquad \text{with } L = \int_\gamma 1\,dl.$$

Find the center of mass of:
(a) one segment of the cycloid;
(b) semi-cicle $x(t) = \cos t$, $y(t) = \sin t$, $t \in [0, \pi]$.

(6) Evaluate $\displaystyle\int_\gamma P(x, y)dx + Q(x, y)dy$ if

(a) $P(x, y) = x^2 + y^2$, $Q(x, y) = xy$ and $\gamma$ is a line segment from $(-1, 1)$ to $(1, 0)$;
(b) $P(x, y) = -y$, $Q(x, y) = x$ and $\gamma$ is the unit circle, oriented counter-clockwise;

(7) Evaluate $\displaystyle\int_\gamma P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz$ if

(a) $P(x, y, z) = y$, $Q(x, y, z) = z$, $R(x, y, z) = x$ and $\gamma$ is a segment of the helix $x(t) = \cos t$, $y(t) = \sin t$, $z(t) = t$, $t \in [0, 2\pi]$;
(b) $P(x, y, z) = z$, $Q(x, y, z) = y$, $R(x, y, z) = x$ and $\gamma$ is given by $x(t) = t + e^t$, $y(t) = t - e^t$, $z(t) = te^t$, $t \in [0, 1]$.