Practice problems #6: double integrals

(1) Find double integrals:

(a) $\iint_D \frac{1}{(x+y+1)^3}\,dx\,dy$, where $D = [0,1] \times [1,2]$;

(b) $\iint_D x\sin(xy)\,dx\,dy$, where $D = [0,1] \times [\pi, 2\pi]$.

(2) Express the double integral $\iint_D f(x,y)\,dx\,dy$ as iterated integrals, if:

(a) D is the region bounded by the curves $y = x^2$ and $x = y^2$;

(b) D is the region bounded by the curves $x^2 - y^2 = 1$ and $x = y^2$.

(3) Interchange the order of integration in the iterated integrals:

(a) $\int_0^1 dx \int_0^x f(x,y)dy$;

(b) $\int_{-1}^1 dx \int_0^{1-x^2} f(x,y)dy$;

(c) $\int_{-2}^2 dx \int_{-\sqrt{8-2x^2}}^{\sqrt{8-2x^2}} f(x,y)dy$.

(4) Interchange the order of integration and then evaluate $\int_0^3 dy \int_1^{\sqrt{4-y}} (x+y)dx$.

(5) Find double integrals:

(a) $\iint_D (xy + 4x^2)\,dx\,dy$, where D is bounded by $y = x+3$ and $y = x^2 + 3x + 3$;

(b) $\iint_D \frac{x}{x^2+y^2}\,dx\,dy$, where D is bounded by $x = 2$, $y = x$ and $y = \frac{1}{2}x$.

(6) Use polar coordinates to find

(a) $\iint_D e^{-x^2-y^2}\,dx\,dy$, where D is a disk centred at $(0,0)$ with radius r;

(b) $\iint_D (1 - \sqrt{x^2+y^2})\,dx\,dy$, where D is the unit disk;

(c) $\iint_D (x^2+y^2)\,dx\,dy$, where $D = \{(x,y) : 0 \leqslant y \leqslant x^2 + y^2 \leqslant x\}$;

(d) $\iint_D 1\,dx\,dy$, where $D = \{x^2 + y^2 \leqslant 2y, y \geqslant \sqrt{3}x\}$;

(e) $\iint_D 1\,dx\,dy$, where $D = \{(x,y) : x^2 + y^2 \leqslant (\frac{\pi}{2})^2 - (\operatorname{arctg}\frac{y}{x})^2\}$.

(7) Use Green's theorem to evaluate

(a) $\oint_\gamma 3xy(x+y)dx + (x^3+y^3)dy$, where γ is the unit circle;

(b) $\oint_D \sin(x+y)dx + \cos(x-y)dy$, where D is the square with vertices at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$.

Assume that γ is oriented counter-clockwise.

(8) Use Green's theorem to argue that the area of the region bounded by a simple closed curve γ (oriented counter-clockwise) is equal to

$$\oint_\gamma 0dx + xdy = -\oint_\gamma ydx + 0dy.$$