Practice problems #7: first order ODEs

(1) Half-life of polonium-210 (undergoing radioactive decay) is 140 days. Knowing that the initial amount of this radioactive isotope is $y(0) = 1$ unit and that $y'(t)$ is proportional to $y(t)$, find $y(t)$.

(2) Find general solutions of

   (a) $yy' + 4t = 0,$ (b) $dy = 2ty^2 dt,$ (c) $t(y^2 - 1)dt + y(t^2 - 1)dy = 0,$

   (d) $2\sqrt{t}y' = \sqrt{1 - y^2},$ (e) $y' = 1 + t + y + ty,$ (f) $y' + 4y = y(e^{-t} + 4).$

(3) Solve initial problems

   (a) $y' \sin t = y \ln y,$ $y(\frac{\pi}{2}) = e,$

   (b) $t\sqrt{1 - y^2}dt + y\sqrt{1 - t^2}dy = 0,$ $y(0) = 1,$

   (c) $t(y + 1)y' = y,$ $y(e) = 1,$

   (d) $y \cos t \, dt - (1 + y^2)dy = 0,$ $y(0) = 1,$

   (e) $y' = y^2(1 + t^2),$ $y(0) = -2,$

   (f) $e^y(y' - 1) = 1,$ $y(0) = 0.$

(4) Find general solutions of

   (a) $y' + y = \sin t,$ (b) $y' + 2ty = e^{-t^2},$ (c) $ty' - 2y = t^3 \cos t,$

   (d) $ty' - 2y = 4t^4,$ (e) $ty + e^t - ty' = 0,$ (f) $(2t + 1)y' = 4t + 2y.$

(5) Solve initial problems

   (a) $y' - y = 1,$ $y(3) = 3,$

   (b) $y' = (y + 1)\sin t,$ $y(t_0) = y_0,$

   (c) $ty' + y = t + 1,$ $y(1) = 0,$

   (d) $y' \sin t \cos t = y + \sin^3 t,$ $y(\frac{\pi}{4}) = 0.$

(6) Solve $t^2 y' + y = (t^2 + 1)e^t$, knowing that $\lim\limits_{t \to -\infty} y(t) = 1.$

You may use a computer for integration.