Practice problems #8: higher order linear ODEs and systems of linear ODEs

(1) Solve the following ODEs:

(a) $y'' = 2y' - 5y$,

(b) $6y'' - 5y' + y = 0$,

(c) $2y'' - 5y' + 2y = 0$,

(d) $y''' - y'' + 3y' + 5 = 0$,

(e) $y'''' = y$,

(f) $y'''' = -y$.

(2) Solve the initial value problems:

(a) $\begin{cases} y''' = y', \\ y(0) = 1, \\ y'(0) = -1, \\ y''(0) = 0; \end{cases}$

(b) $\begin{cases} y''' = -y', \\ y(0) = 1, \\ y'(0) = -1, \\ y''(0) = 0; \end{cases}$

(c) $\begin{cases} y''' = -y, \\ y(0) = 1, \\ y'(0) = -1, \\ y''(0) = 0. \end{cases}$

(3) For what values of the parameter p does the following *boundary value problem*:

$$\begin{cases} y'' = py, \\ y(0) = 0, \\ y(1) = 0 \end{cases}$$

have a non-zero solution?

(4) Solve the following ODEs:

(a) $y'' - 7y' + 10y = e^{3t}$,

(b) $y'' + 4y = \dfrac{1}{\cos(2t)}$,

(c) $y'' + 3y' + 2y = \dfrac{1}{1 + e^t}$,

(d) $y'''' = y + \cos t$.

(5) Find the general solution for $x(t)$ if

(a) $\begin{cases} x'(t) = x(t) + 3y(t), \\ y'(t) = -x(t) + 5y(t); \end{cases}$

(b) $\begin{cases} x'(t) = 3x(t) - 2y(t), \\ y'(t) = 4x(t) + 7y(t); \end{cases}$

(c) $\begin{cases} x'(t) = 2x(t) - y(t) + z(t), \\ y'(t) = x(t) + 2y(t) - z(t), \\ z'(t) = x(t) - y(t) + 2z(t). \end{cases}$

(6) Find $x(t)$ and $y(t)$ if

(a) $\begin{cases} x'(t) = x(t) + 3y(t), \quad x(0) = 3, \\ y'(t) = -x(t) + 5y(t), \quad y(0) = 1; \end{cases}$

(b) $\begin{cases} x'(t) = 3x(t) - 2y(t), \quad x(0) = 1, \\ y'(t) = 4x(t) + 7y(t), \quad y(0) = 0. \end{cases}$