Practice problems #9: Laplace transform

(1) Find Laplace transforms of the functions
$$f(t) = \begin{cases} 1 & \text{if } t \in [0, 1], \\ 0 & \text{otherwise}; \end{cases}$$

$$g(t) = \begin{cases} 1 - t & \text{if } t \in [0, 1], \\ 0 & \text{otherwise}; \end{cases}$$

$$h(t) = \begin{cases} 0 & \text{if } t \in [0, 1], \\ 1 & \text{otherwise}. \end{cases}$$

(2) Find general solutions of the ODEs
$$\text{(a) } y'' + 4y' + 4y = 0, \quad \text{(b) } y'''' - 2y''' + 2y' - y = 0, \quad \text{(c) } y'''' + 2y'' + y = 0.$$

(3) Solve once more the following ODEs:
$$\text{(a) } y'' - 7y' + 10y = e^{3t}, \qquad\qquad\qquad \text{(d) } y'''' = y + \cos t .$$

(4) Find $x(t)$ and $y(t)$ if
$$\text{(a) } \begin{cases} x'(t) = x(t) + 3y(t) + e^{-t}, & x(0) = 3, \\ y'(t) = -x(t) + 5y(t) - e^{-2t}, & y(0) = 1; \end{cases}$$

$$\text{(b) } \begin{cases} x'(t) = 3x(t) - 2y(t) + t, & x(0) = 1, \\ y'(t) = 4x(t) + 7y(t) - t^2, & y(0) = 0. \end{cases}$$