This list is prepared for the students who will intend to achieve the highest possible grade - Excellent. It will contain advanced tasks that will often require some knowledge beyond the standard programme and, therefore, extra materials may be necessary. To obtain the desired grade, the candidate must complete all the tasks on separate sheets of paper and give them to the teacher for correction. If the solutions are accepted, the candidate will present them during a lesson or extra consultations.

Note that to achieve the grade 'Excellent' it is necessary to satisfy the conditions for the grade 'Very Good'.

Tasks for 32IB - 1st semester

1. a) What is the common feature of the functions: $\sin x, 5x, 4x^7 - 5x^3 + 8x, \tan(2x), \frac{1}{x^3}$? Find their derivatives. What do they have in common?
   b) Repeat the steps above for the functions $3 + \cos x, 3x^2 + 5, \sqrt[3]{2x^6 - 3x^4 + 5}, \sqrt{1 + |x|}$.
   c) Generalize your findings and prove them.
   d) Investigate the same properties for integration.
2. Integrals of the 2nd order partial fractions.
   Let $a \neq 0,\ b, p \in R,\ q > 0$ and $n \in N_+$.
   a) Derive the formula for $\displaystyle\int \frac{ax + b}{(x - p)^2 + q} dx$.
   b) Using $\displaystyle\int \frac{1}{x^2 + 1} dx$ find, directly, $\displaystyle\int \frac{1}{(x^2 + 1)^2} dx$.
   c) Let $I_n = \displaystyle\int \frac{1}{(x^2 + 1)^n} dx$. Generate a recursive formula that relates $I_n$ to $I_{n-1}$.
   d) Find a similar formula for $\displaystyle\int \frac{1}{((x - p)^2 + q)^n} dx$ and, next, for $\displaystyle\int \frac{ax + b}{((x - p)^2 + q)^n} dx$.
3. a) State the definition of a definite integral as a limit of the $n$'th integral sum.
   b) Show that if $f$ is continuous in $[a, b]$ then this definition implies the Leibnitz - Newton's definition of integral.
4. Find, exactly, the following limits.
   a) $\displaystyle\lim_{n\to\infty} \frac{1^7 + 2^7 + ... + n^7}{n^8}$
   d) $\displaystyle\lim_{n\to\infty} \frac{n}{n^2 + n + 1^2} + \frac{n}{n^2 + 2n + 2^2} + \frac{n}{n^2 + 3n + 3^2} ... + \frac{n}{n^2 + n^2 + n^2}$
   e) $\displaystyle\lim_{n\to\infty} \ln \frac{\sqrt[n]{(n + 1)(n + 2)...(n + n)}}{n}$