Transformations of graphs

1. Let $f(x) = \sqrt{x}$. Write the formula of a function g whose graph is a result of the transformation listed below, applied to the graph of f. You may sketch the graphs of f and g to confirm your results. Also, note down in which cases the results are the same.

    1. Translation 3 units leftwards, followed by translation 2 units rightwards.
    2. Translation 2 units rightwards, followed by translation 3 units leftwards.
    3. Translation 3 units upwards, followed by translation 2 units downwards.
    4. Translation 2 units downwards, followed by translation 3 units upwards.
    5. Translation 3 units rightwards, followed by translation 2 units upwards.
    6. Translation 2 units upwards, followed by translation 3 units rightwards.

    7. Translation 3 units leftwards, followed by reflection about the X-axis.
    8. Reflection about the X-axis, followed by translation 3 units leftwards.
    9. Translation 3 units rightwards, followed by reflection about the Y-axis.
    10. Reflection about the Y-axis, followed by translation 3 units rightwards.
    11. Translation 3 units upwards, followed by reflection about the X-axis.
    12. Reflection about the X-axis, followed by translation 3 units upwards.
    13. Translation 3 units downwards, followed by reflection about the Y-axis.
    14. Reflection about the Y-axis, followed by translation 3 units downwards.

    15. Translation 3 units leftwards, followed by enlargement of factor 2 from the X-axis.
    16. Enlargement of factor 2 from the X-axis, followed by translation 3 units leftwards.
    17. Translation 3 units rightwards, followed by enlargement of factor 2 from the Y-axis.
    18. Enlargement of factor 2 from the Y-axis, followed by translation 3 units rightwards.
    19. Translation 3 units upwards, followed by enlargement of factor 2 from the X-axis.
    20. Enlargement of factor 2 from the X-axis, followed by translation 3 units upwards.
    21. Translation 3 units downwards, followed by enlargement of factor 2 from the Y-axis.
    22. Enlargement of factor 2 from the Y-axis, followed by translation 3 units downwards.

    23. Reflection about the X-axis, followed by reflection about the X-axis.
    24. Reflection about the Y-axis, followed by reflection about the Y-axis.
    25. Reflection about the X-axis, followed by reflection about the Y-axis.
    26. Reflection about the Y-axis, followed by reflection about the X-axis.

    27. Reflection about the X-axis, followed by enlargement of factor 0.5 from the X-axis.
    28. Enlargement of factor 0.5 from the X-axis, followed by reflection about the X-axis.
    29. Reflection about the X-axis, followed by enlargement of factor 0.5 from the Y-axis.
    30. Enlargement of factor 0.5 from the Y-axis, followed by reflection about the X-axis.
    31. Reflection about the Y-axis, followed by enlargement of factor 0.5 from the X-axis.
    32. Enlargement of factor 0.5 from the X-axis, followed by reflection about the Y-axis.

33. Reflection about the Y-axis, followed by enlargement of factor 0.5 from the Y-axis.
34. Enlargement of factor 0.5 from the Y-axis, followed by reflection about the Y-axis.

35. Enlargement of factor 0.5 from the X-axis, followed by enlargement of factor 4 from the X-axis.
36. Enlargement of factor 4 from the X-axis, followed by enlargement of factor 0.5 from the X-axis.
37. Enlargement of factor 0.5 from the X-axis, followed by enlargement of factor 4 from the Y-axis.
38. Enlargement of factor 4 from the Y-axis, followed by enlargement of factor 0.5 from the X-axis.
39. Enlargement of factor 0.5 from the Y-axis, followed by enlargement of factor 4 from the Y-axis.
40. Enlargement of factor 4 from the Y-axis, followed by enlargement of factor 0.5 from the Y-axis.

2. Solve task 1 for $f(x) = x^3 - 3x + 2$ and the following transformations.

    1. Translation 1 unit leftwards, followed by translation 4 units rightwards.
    2. Translation 4 units rightwards, followed by translation 1 unit leftwards.
    3. Translation 1 unit upwards, followed by translation 4 units downwards.
    4. Translation 4 units downwards, followed by translation 1 unit upwards.
    5. Translation 1 unit rightwards, followed by translation 4 units upwards.
    6. Translation 4 units upwards, followed by translation 1 unit rightwards.

    7. Translation 1 unit leftwards, followed by reflection about the X-axis.
    8. Reflection about the X-axis, followed by translation 1 unit leftwards.
    9. Translation 1 unit rightwards, followed by reflection about the Y-axis.
    10. Reflection about the Y-axis, followed by translation 1 unit rightwards.
    11. Translation 1 unit upwards, followed by reflection about the X-axis.
    12. Reflection about the X-axis, followed by translation 1 unit upwards.
    13. Translation 1 unit downwards, followed by reflection about the Y-axis.
    14. Reflection about the Y-axis, followed by translation 1 unit downwards.

    15. Translation 1 unit leftwards, followed by enlargement of factor 3 from the X-axis.
    16. Enlargement of factor 3 from the X-axis, followed by translation 1 unit leftwards.
    17. Translation 1 unit rightwards, followed by enlargement of factor 3 from the Y-axis.
    18. Enlargement of factor 3 from the Y-axis, followed by translation 1 unit rightwards.
    19. Translation 1 unit upwards, followed by enlargement of factor 3 from the X-axis.
    20. Enlargement of factor 3 from the X-axis, followed by translation 1 unit upwards.
    21. Translation 1 unit downwards, followed by enlargement of factor 3 from the Y-axis.
    22. Enlargement of factor 3 from the Y-axis, followed by translation 1 unit downwards.

    23. Reflection about the X-axis, followed by reflection about the X-axis.
    24. Reflection about the Y-axis, followed by reflection about the Y-axis.
    25. Reflection about the X-axis, followed by reflection about the Y-axis.
    26. Reflection about the Y-axis, followed by reflection about the X-axis.

27. Reflection about the X-axis, followed by enlargement of factor 2.5 from the X-axis.
28. Enlargement of factor 2.5 from the X-axis, followed by reflection about the X-axis.
29. Reflection about the X-axis, followed by enlargement of factor 2.5 from the Y-axis.
30. Enlargement of factor 2.5 from the Y-axis, followed by reflection about the X-axis.
31. Reflection about the Y-axis, followed by enlargement of factor 2.5 from the X-axis.
32. Enlargement of factor 2.5 from the X-axis, followed by reflection about the Y-axis.
33. Reflection about the Y-axis, followed by enlargement of factor 2.5 from the Y-axis.
34. Enlargement of factor 2.5 from the Y-axis, followed by reflection about the Y-axis.

35. Enlargement of factor 0.4 from the X-axis, followed by enlargement of factor 2 from the X-axis.
36. Enlargement of factor 2 from the X-axis, followed by enlargement of factor 0.4 from the X-axis.
37. Enlargement of factor 0.4 from the X-axis, followed by enlargement of factor 2 from the Y-axis.
38. Enlargement of factor 2 from the Y-axis, followed by enlargement of factor 0.4 from the X-axis.
39. Enlargement of factor 0.4 from the Y-axis, followed by enlargement of factor 2 from the Y-axis.
40. Enlargement of factor 2 from the Y-axis, followed by enlargement of factor 0.4 from the Y-axis.

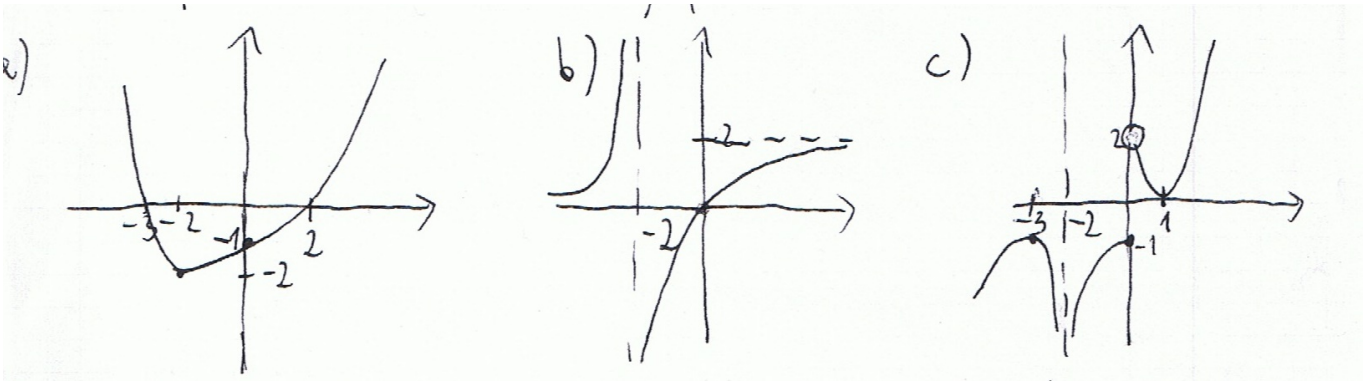
3. Solve task 1 for $f(x) = \sqrt{x}$ and the following transformations.

    1. Translation 3 units rightwards, followed by translation 2 units upwards, then followed by reflection about the X-axis.
    2. Translation 3 units rightwards, followed by reflection about the X-axis, then followed by translation 2 units upwards.
    3. Translation 2 units upwards, followed by translation 3 units rightwards, then followed by reflection about the X-axis.
    4. Translation 2 units upwards, followed by reflection about the X-axis, then followed by translation 3 units rightwards.
    5. Reflection about the X-axis, followed by translation 3 units rightwards, then followed by translation 2 units upwards.
    6. Reflection about the X-axis, followed by translation 2 units upwards, then followed by translation 3 units rightwards.
    7. Reflection about the Y-axis, followed by enlargement of factor 2 from the Y-axis, then followed by translation 2 units downwards.
    8. Reflection about the Y-axis, followed by translation 2 units downwards, then followed by enlargement of factor 2 from the Y-axis.
    9. Translation 2 units downwards, followed by enlargement of factor 2 from the Y-axis, then followed by reflection about the Y-axis.
    10. Translation 2 units leftwards, followed by enlargement of factor 0.5 from the X-axis, then followed by reflection about the Y-axis, then followed by translation 1 unit upwards.
    11. Enlargement of factor 0.5 from the X-axis, followed by translation 2 units leftwards, then followed by translation 1 unit upwards, then followed by reflection about the Y-axis.
    12. Translation 3 units leftwards, followed by enlargement of factor 0.5 from the X-axis, then followed by reflection about the X-axis, then followed by enlargement of factor 4 from the Y-axis, then followed by reflection about the Y-axis, then followed by translation 1 unit upwards.

4. Let $f(x) = \sqrt{x}$. For the formulas of g given below state the transformations that, applied to the graph of f, give the graph of g. State, precisely, the order of these transformations.

   1. $g(x) = \sqrt{0.5x} + 3$.
   2. $g(x) = \sqrt{0.5x - 3}$.
   3. $g(x) = \sqrt{3 - x}$.
   4. $g(x) = \sqrt{-x} + 3$.
   5. $g(x) = 2\sqrt{x} - 3$.
   6. $g(x) = 2\sqrt{x - 3}$.
   7. $g(x) = -\sqrt{x} - 3$.
   8. $g(x) = -\sqrt{x - 3}$.
   9. $g(x) = \sqrt{2x + 1}$.
   10. $g(x) = \sqrt{2(x + 1)}$.
   11. $g(x) = -2\sqrt{x} - 2$.
   12. $g(x) = -4\sqrt{-2x + 3} + 1$.

5. The graphs of f are given below. Sketch the graphs of $|f(x)|$, $f(|x|)$, $|f(|x|)|$, $f^2(x)$ and $\dfrac{1}{f(x)}$. Indicate clearly all characteristic points and the asymptotes.



6. The graph of f is given below. Sketch the graphs of

   1. $g(x) = 2f(x)$,
   2. $g(x) = f(1 - x)$,
   3. $g(x) = f(2x) + 1$,
   4. $g(x) = -f(x + 1)$,
   5. $g(x) = f(|x|) + 1$,
   6. $g(x) = \left| f\left(\dfrac{1}{2}x\right) \right|$,
   7. $g(x) = f(|x| - 1)$,
   8. $g(x) = |f(x) + 3|$,
   9. $g(x) = \dfrac{1}{f(x - 1)}$,
   10. $g(x) = \dfrac{2}{f(x)} + 1$,

11. $g(x) = f^2(x) - 1$,
12. $g(x) = (f(x) + 1)^2$.

Indicate clearly all characteristic points and the asymptotes.