

Linear equations

1. Solve the systems below.

$$\text{a) } \begin{cases} x + 2y + z = -1 \\ x + y + 3z = 2 \\ x + 3y + 8z = 1 \end{cases} \quad \text{b) } \begin{cases} x + 2y + z = -1 \\ x + y + 3z = 2 \\ 2x + 3y + 4z = 6 \end{cases} \quad \text{c) } \begin{cases} x + 2y + z = -1 \\ x + y + 3z = 2 \\ 2x + 3y + 4z = 1 \end{cases}$$

$$\text{d) } \begin{cases} x + 2y + z = -1 \\ x + y + 3z = 2 \end{cases} \quad \text{e) } \begin{cases} x - 2y + z = -4 \\ x + y + z = 1 \\ 2x - 3y + 5z = 10 \\ 5x - 6y + 18z = 19 \end{cases} \quad \text{f) } \begin{cases} x + 2y + z + t = 7 \\ 2x - y - z + 4t = 2 \\ 5x + 5y + 2z + 7t = 1 \end{cases}$$

$$\text{g) } \begin{cases} x + 2y + z + t = 7 \\ 2x - y - z + 4t = -20 \\ 5x + 5y + 2z + 7t = 1 \end{cases} \quad \text{h) } \begin{cases} x - 2y + z - t = -4 \\ 2x - y - z + t = 1 \\ x + y + 2z - t = 5 \\ x + y - z + t = 4 \end{cases} \quad \text{i) } \begin{cases} x + 2y + 3z + t = 1 \\ 2x + 4y - z + 2t = 2 \\ 3x + 6y + 10z + 3t = 3 \\ x + y + z + t = 0 \end{cases}$$

$$\text{j) } \begin{cases} 3x + y - 2t = 1 \\ 5x + 2y + 2z - t = 5 \\ x - y - 2t = -5 \\ 5x + y + z - 3t = 0 \\ -7x - 3y + z + 5t = -4 \end{cases} \quad \text{k) } \begin{cases} x - y + z - 2s + t = 0 \\ 3x + 4y - z + s + 3t = 1 \\ x - 8y + 5z - 9s + t = -1 \end{cases} \quad \text{l) } \begin{cases} x - 3y + z - 2s + t = -5 \\ 2x - 6y - 4s + t = -10 \\ 2z + t = 0 \\ -2x + 6y + 2z + 4s = 10 \\ -2x + 6y + 4z + 4s + t = 10 \\ -x + 3y + z + 2s = 5 \end{cases}$$

2. Find all $p \in R$ for which the systems below have unique solutions. For other values of p solve the systems.

$$\text{a) } \begin{cases} x + 2y + z = -1 \\ y + 3z = 2 \\ x + 3y + pz = 0 \end{cases} \quad \text{b) } \begin{cases} (p + 5)x + 2y + 4z = 2 \\ 4x + py + 2z = 2 \\ 3x + y + 2z = 1 \end{cases} \quad \text{c) } \begin{cases} x + 4y - 2z = -p \\ 3x + 5y - pz = 3 \\ px + 3py + z = p \end{cases}$$

3. Using the method of elimination determine for which values of $p \in R$ the systems below are solvable and solve them.

$$\text{a) } \begin{cases} x + y + z = 1 \\ 2x - y + 3z = 5 \\ x - 2y + 2z = p \end{cases} \quad \text{b) } \begin{cases} x + 2y + z = 3 \\ 2x + 3y - 5z = p \\ -x - y + 6z = 1 \end{cases} \quad \text{c) } \begin{cases} x + 2y - 3z = 2 \\ -3x + y - z = p \\ -2x + 3y - 4z = p^2 \end{cases}$$