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MATHEMATICS

Higher Level

The portfolio – tasks

For use in 2011 and 2012

For final assessment in 2011 and 2012

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Introduction

What is the purpose of this document?

This document contains new tasks for the portfolio in mathematics HL. These tasks have been produced by the IB, for teachers to use in 2011 and 2012. It should be noted that any tasks previously produced and published by the IB will no longer be valid for assessment after the November 2010 examination session. These include all the tasks in any teacher support material (TSM), and the tasks in the document "Portfolio tasks 2009–2010". Copies of all TSM tasks published by the IB are available on the Online Curriculum Centre (OCC), under **Internal Assessment**, in a document called "Old tasks published prior to 2008". These tasks should **not** be used, even in slightly modified form.

What happens if teachers use these old tasks?

The inclusion of these old tasks in the portfolio will make the portfolio non-compliant, and such portfolios will therefore attract a 10-mark penalty. Teachers may continue to use the old tasks as practice tasks, but they should not be included in the portfolio for final assessment.

What other documents should I use?

All teachers should have copies of the mathematics HL subject guide (second edition, September 2006), including the teaching notes appendix, and the TSM (September 2005). Further information, including additional notes on applying the criteria, is available on the Online Curriculum Centre (OCC). Important news items are also available on the OCC, as are the diploma programme coordinator notes, which contain updated information on a variety of issues.

Which tasks can I use in 2011?

The only tasks produced by the IB that may be submitted for assessment in 2011 are the ones contained in this document. (These tasks may also be used in 2012, along with the other tasks for 2012 and 2013 that will be published by January 2011.) There is no requirement to use tasks produced by the IB, and there is no date restriction on tasks written by teachers.

Can I use these tasks before May 2011?

These tasks should only be submitted for final assessment from May 2011 to November 2012. Students should not include them in portfolios before May 2011. If they are included, they will be subject to a 10-mark penalty. Please note that these dates refer to examination sessions, not when the work is completed.

PATTERNS WITHIN SYSTEMS OF LINEAR EQUATIONS **HL TYPE I**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

In this task, you will investigate systems of linear equations where the system constants have well known mathematical patterns.

Part A Consider this 2×2 system of linear equations $\begin{cases} x+2y=3 \\ 2x-y=-4. \end{cases}$

- Examine the constants in the first equation and describe any patterns. Repeat for the second equation.
- Solve the above system. Display your solution graphically. What is the significance of the solution?
- Create and solve a few more systems similar to the example above. Comment on your solutions.
- Make a conjecture regarding this type of 2×2 system and prove it.
- Using technology, extend your investigation to 3×3 systems whose constants exhibit the patterns seen above.
- Make a conjecture regarding the solution to those 3×3 systems and prove your conjecture.

Part B Consider this 2×2 system $\begin{cases} x+2y=4 \\ 5x-y=\dfrac{1}{5}. \end{cases}$

- Examine the constants in both equations and describe any patterns.
- Re-write the above two equations in the form $y=ax+b$. How are the constants a and b related?
- Using technology, create a family of linear equations similar to the example shown above. On the same set of axes, display your equations. Ensure that the family of equations includes several lines with a wide range of gradients.
- Closely examine the resulting graphical display. Clearly describe any apparent graphical patterns.
- Set up and solve a general 2×2 system that incorporates the patterns found above.
- Using your solution to the general system, or otherwise, provide a proof of the graphical pattern observed above.

HOW MANY PIECES? **HL TYPE I**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

In this task you will investigate the maximum number of pieces obtained when an *n*-dimensional object is cut.

- A line segment is a finite one-dimensional object. Find a rule which relates the maximum number of segments (*S*) obtained from n cuts. Comment on your result.

- A circle is a finite two-dimensional object. Investigate the maximum number of regions (*R*) obtained when n chords are drawn .

  - Draw sketches to show your answers for $n = 1,\ 2,\ 3,\ 4$ an d5.
  - Tabulate your results.
  - Find a recursive rule to generate the maximum number of regions.
  - Use technology to make a conjecture for the relationship between the maximum number of regions (*R*) and the number of chords (*n*).
  - Prove your conjecture.
  - Rewrite your formula in the form $R = X + S$, where X is an algebraic expression in n.

- A cuboid is a finite three-dimensional object. Investigate the maximum number of parts (*P*) that are obtained with n cuts.

  - Use geometry software to investigate the maximum number of parts for $n = 1,\ 2,\ 3,\ 4$ an d5 (free three-dimensional geometry software is readily available online).
  - Tabulate your results.
  - Find a recursive rule to generate the maximum number of parts.
  - Use technology to make a conjecture for the relationship between the maximum number of parts (*P*) and the number of cuts (*n*).
  - Prove your conjecture.
  - Rewrite your formula in the form $P = Y + X + S$, where Y is an algebraic expression in n.

- If a finite four-dimensional object exists and the procedure is repeated what would you expect to find?

  - Using a spreadsheet and the results of the investigation above, make a conjecture for a recursive formula which gives the maximum number of parts, Q, obtained when n cuts are made on a finite four-dimensional object.
  - Prove your conjecture.
  - Rewrite your formula as $Q = Z + Y + X + S$.
  - Comment on your results.

RUNNING WITH ANGIE AND BUDDY **HL TYPE II**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

In this task, you will model the route traced by a pet dog named Buddy as he follows his owner Angie.

Consider a situation in which Angie runs in the North direction along the straight edge of a lake. Buddy starts out d metres to the East of Angie as illustrated in figure 1 below. Both runners start from rest.

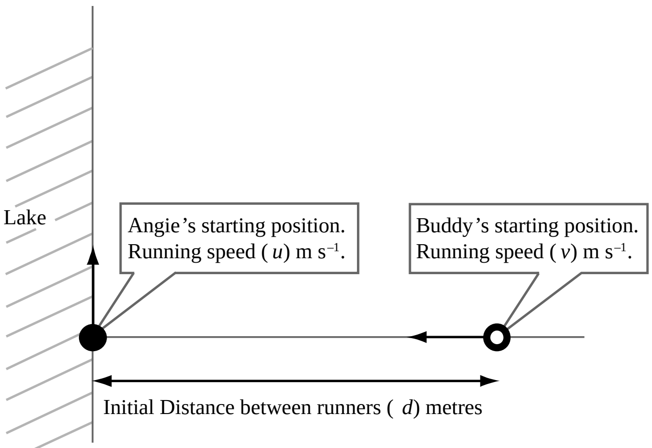

Figure 1

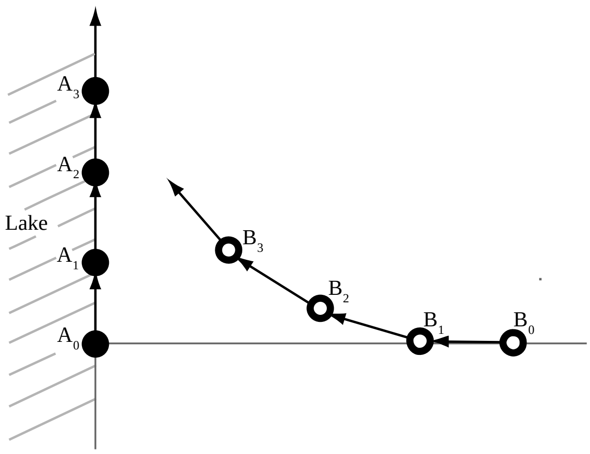

Figure 2

Angie runs at a constant speed of u m s^{-1}. Buddy notices Angie and runs towards her at a constant speed v m s^{-1}. Buddy runs directly towards where Angie is in an attempt to catch her. He changes direction when he looks up to notice Angie's new position as shown in figure 2. Buddy does not like to get wet and will avoid taking a plunge into the lake.

- Choose reasonable values for Angie's running speed u m s^{-1} and Buddy's running speed v m s^{-1}. Fully justify your choices for these values.
- Your teacher will supply you with information on:
  - the initial distance between the runners.
  - how often Buddy looks up to notice Angie's position and hence adjusts his own direction (*e.g.* Buddy changes direction every t seconds).
- Construct a diagram to plot the positions and directions of both runners. Use trigonometry to find the coordinates of several points where Buddy changes his direction.
- Determine recursive formulae for the x- and y-coordinates of Buddy's position to establish a discrete mathematical model.

(This task continues on the following page)

- Using a spreadsheet, or geometric software, simulate the positions of the two runners for a variety of values of the speeds u and v, the time t and the distance d.

- Using the data generated above, discuss the condition(s) necessary for Buddy to catch up with Angie. Again, include estimates for:

  - the time at which Buddy catches up with Angie.
  - the distance travelled by each runner when Buddy catches up.

- Discuss the assumptions in the above model. What limitations does it have when applied to other situations?

MODELLING A FUNCTIONAL BUILDING **HL TYPE II**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

As an architect you have been contracted to design a building with a roof structure similar to the one shown below.

In this task you are to model such a building and produce a report to the contractor with all necessary specifications.

(This task continues on the following page)

You are to design an office block inside a structure with the following specifications:

The building has a rectangular base 150 m long and 72 m wide. The maximum height of the structure should not exceed 75 % of its width for stability or be less than half the width for aesthetic purposes. The minimum height of a room in a public building is 2.5 m.

- Create a model for the curved roof structure when the height is 36 m.
- Find the dimensions of the cuboid with maximum volume which would fit inside this roof structure.
- Use technology to investigate how changes to the height of the structure affect the dimensions of the largest possible cuboid.
- For each height, calculate the ratio of the volume of the wasted space to the volume of the office block.
- Determine the total maximum office floor area in the block for different values of height within the given specifications.
- Given that the base remains the same (72 m $\times$ 150 m), investigate what would happen if the facade is placed on the longer side of the base.
- You now decide to maximise office space even further by not having the block in the shape of a single cuboid.
- Review your model and calculate the increase in floor area and the new volume ratio of wasted space to office block.