MATHEMATICAL INDUCTION "DOMINO EFFECT"

1) IF NATURAL NUMBER n_0 SATISFIES A PROPERTY (A)

2) IF FOR ANY $n \geq n_0$ WE CAN SHOW THAT FROM n HAVING (A) IT FOLLOWS THAT $n+1$ ALSO HAS PROP (A)

THEN IT FOLLOWS THAT ALL NUMBERS $n \geq n_0$ HAVE PROP (A).

1) IS CALLED BEGGINING OF THE INDUCTION

IND. STEP 2) ASSUMPTION THAT (A) IS TRUE FOR (n) IS CALLED INDUCTION ASSUMPTION AND QUESTION IF (A) IS TRUE FOR $n+1$ IS INDUCTION HYPOTHESIS

EX 1 BERNOULLI INEQUALITY

FOR ALL $n \in \mathbb{N}_+$ $(1+a)^n \geq 1 + n \cdot a$ FOR $a > -1$

- IS IT TRUE FOR $n=1$? BEGGINIG OF INDUCTION

1) $(1+a)^1 \geq 1 + 1 \cdot a$ $0 \geq 0$

$\quad = \qquad\qquad = \qquad$ TRUE ✓

$\quad 1+a \qquad 1+a$

- ASSUME THAT INEQ. IS TRUE FOR n:

$(1+a)^n \geq 1 + n a$ INDUCTION ASSUMPTION

2) $\rightarrow$ IT'S ALSO TRUE FOR $n+1$;

WTS; $\Rightarrow (1+a)^{n+1} \geq 1 + (n+1) \cdot a$ ← INDUCTION HYPHOTHESIS

$$(1+a)^n \geq 1 + na \quad / \cdot (1+a)$$

$$(1+a)^{n+1} \geq 1 + na + a + na^2$$

$$(1+a)^{n+1} \geq 1 + (n+1)\cdot a + \underbrace{na^2}_{\geq 0} \geq 1 + (n+1)\cdot a$$

$$(1+a)^{n+1} \geq 1 + (n+1)\cdot a$$

BUT THIS IS WHAT WE WANTED TO SHOW

INDUCTION STEP IS PROVED

BY MATHEMATICAL INDUCTION BERNULLI INEQ. IS TRUE FOR ALL NATURAL NUMBERS n.

EX 2

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \quad \text{FOR } n \in \mathbb{N}$$

PROOF:

1) BEGGINING OF THE INDUCTION: IS IT TRUE FOR $n = 1$

$$1 = \frac{1^3}{3} + \frac{1^2}{2} + \frac{1}{6}$$

$$1 = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{2+3+1}{6} = \frac{6}{6}$$

$1 = 1$ **TRUE**

2) INDUCTION STEP

WE KNOW

INDUCTION ~~HYPOTHESIS~~ ASUMPTION

$$\underbrace{1^2 + 2^2 + 3^2 + \dots + n^2}_{*} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

INDUCTION HYPOTHESIS

$$1^2+2^2+\dots n^2+(n+1)^2 = \frac{(n+1)^3}{3}+\frac{(n+1)^2}{2}+\frac{n+1}{6}$$

TRYING TO SHOW

$$\underbrace{1^2+2^2+\dots n^2}_{*}+(n+1)^2$$

$$(n+1)^2 = \frac{(n+1)^3}{3}+\frac{(n+1)^2}{2}+\frac{n+1}{6}-\frac{n^3}{3}-\frac{n^2}{2}-\frac{n}{6}$$

$$\frac{(n+1)^3-n^3}{3} = \frac{\cancel{n^3}-\cancel{n^3}+3n^2+3n+1}{3} = n^2+n+\frac{1}{3}$$

$$\frac{(n+1)^2-n^2}{2} = \frac{\cancel{n^2}-\cancel{n^2}+2n+1}{2} = n+\frac{1}{2}$$

$$(n+1)^2 = n^2 + n + \frac{1}{3} + n + \frac{1}{2} + \frac{1}{6} = n^2 + 2n + 1$$

$$(n+1)^2 \stackrel{?}{=} n^2 + 2n + 1 \quad \checkmark$$

TRUE, WHICH PROVES INDUCTION STED

THUS, BY MATHEMATICAL INDUCTION WE CAN CONCLUDE:

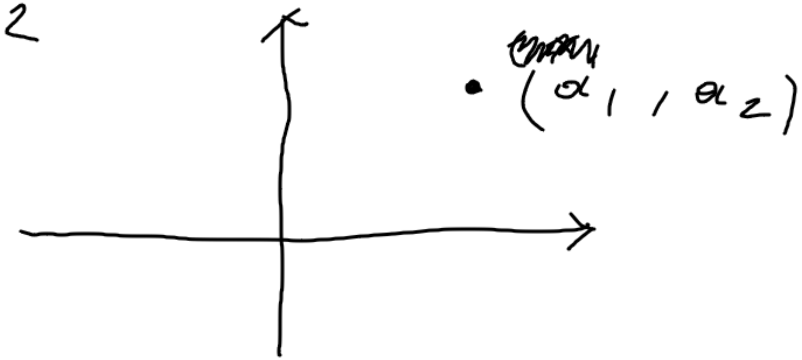
$$1^2 + \dots n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

EUCLIDEAN SPACE $\mathbb{R}^n$

$n =$ dimension

$n = 1$

$n = 2$



$n = 3$

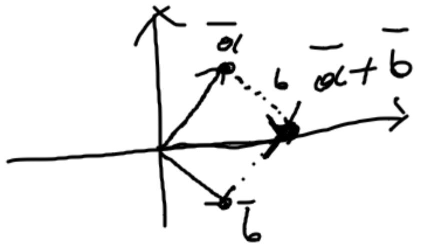
(a_1, a_2, a_3, a_4)
$\in \mathbb{R}^4$

$(a_1, a_2, a_3, \dots a_n) \in \mathbb{R}^n$

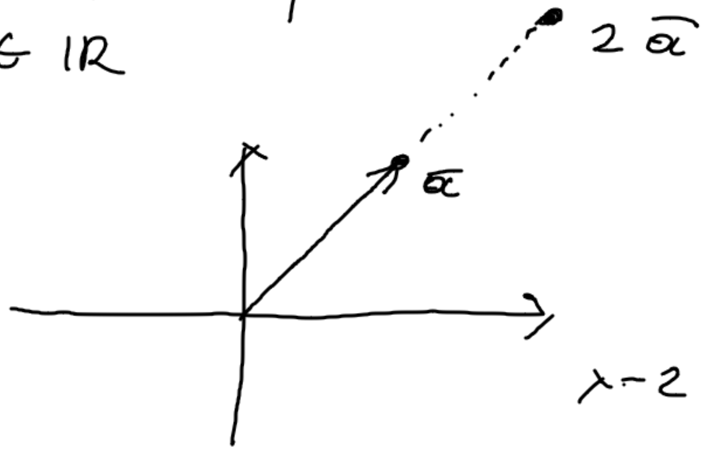
VECTOR SPACES (A.K.A. LINEAR SPACES) OVER REAL NUMBERS, WHICH MEANS WE HAVE SOME OPERATIONS OF VECTORS (POINTS IN THE SPACE)

$\bar{a}, \bar{b} \in \mathbb{R}^n$, $\quad \lambda, \mu \in \mathbb{R}$

- $\bar{a} + \bar{b} \in \mathbb{R}^n$



- $\lambda \bar{a}$



SOME PROPERTIES:

- $\bar{a} + \bar{b} = \bar{b} + \bar{a}$
- $(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$
- $\bar{a} + 0 = \bar{a}$
- $\bar{a} + (-1) \cdot \bar{a} = \bar{0}$
- $1 \cdot \bar{a} = \bar{a}$
- $\lambda(\mu \bar{a}) = (\lambda \cdot \mu)\bar{a}$
- $\lambda(\bar{a} + \bar{b}) = \lambda \bar{a} + \lambda \bar{b}$
- $(\lambda + \mu)\bar{a} = \lambda \bar{a} + \mu \bar{a}$

$\overline{a} = (a_1, a_2 \dots a_n)$ $\lambda \in \mathbb{R}$

$\overline{b} = (b_1, b_2 \dots b_n)$

$\overline{a} + \overline{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, \dots, a_n + b_n)$

<u>EX</u> $(1, 2)$ $(4, 6)$

$n = 2$ $\cancel{}$ $(1, 2) + (4, 6) = (1 + 4, 2 + 6) = (5, 8)$

$\lambda \overline{a} = (\lambda a_1, \lambda a_2, \dots \lambda a_n)$

<u>EX</u> $n = 2$ $(1, 2)$ $\lambda = 3$

$3 \cdot (1, 2) = (3 \cdot 1, 3 \cdot 2) = (3, 6)$

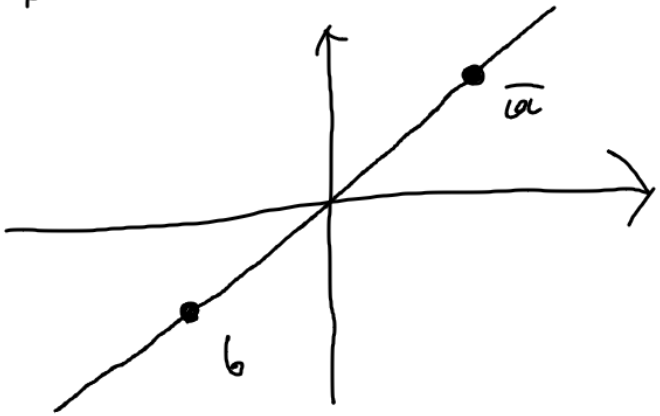
LINES AND SEGMENTS IN $\mathbb{R}^n$

$n = 2$

$$y = a x + b$$

EQUATION FOR A (AFFINE) LINE IN $\mathbb{R}^2$

IN DIMENSION n WE WOULD NEED $n-1$ EQUATIONS TO FOLLOW THIS APROACH.
NOT VERY EFFICIENT IN HIGHER dim.



LINE AS POINTS WHICH ARE A LINEAR COMBINATION OF $\overline{a}$ AND $\overline{b}$ i.e.

$$\{\lambda \cdot \overline{a} + (1-\lambda)\cdot \overline{b}\}$$

$$\lambda \in \mathbb{R}$$

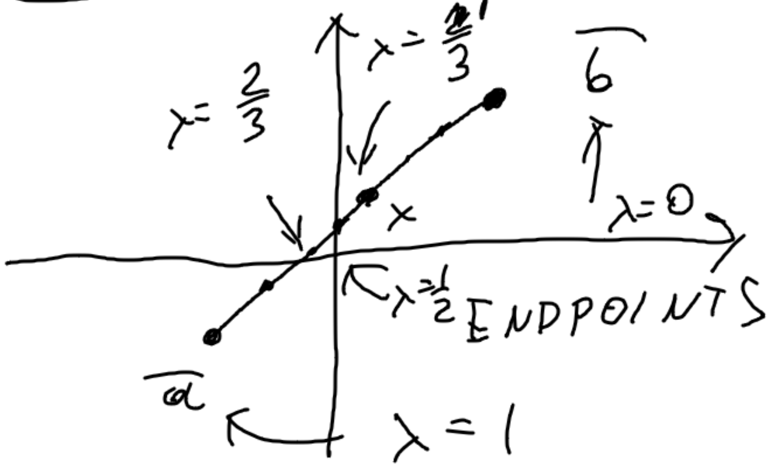
THIS APPROACH WORKS IN $\mathbb{R}^n$:

$(a_1, a_2 \ldots a_n)$, $(b_1 \ldots, b_n) \in \mathbb{R}^n$

THEN $\{ \lambda (a_1 \ldots a_n) + (1-\lambda)(b_1 \ldots b_n) \}$

IS A LINE THROUGH $\bar{a}$ AND $\bar{b}$ IN $\mathbb{R}^n$

SEGMENTS OR INTERVALS



SO INTERVAL IS A SUBSET OF A LINE SO ALGEBRAICLY AN INTERVAL SHOULD ALSO SATISFY LINE EQUATION:

$$\lambda \bar{a} + (1-\lambda) \bar{b}$$

IF WE TAKE $\lambda = 1$ IN $\lambda \bar{a} + (1-\lambda)\cdot \bar{b}$

$$1 \cdot \bar{a} + (1-1)\cdot \bar{b} = \bar{a} + 0 \cdot \bar{b} = \bar{a}$$

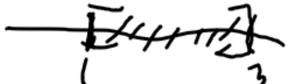
$\lambda = 0$ $$0 \cdot \bar{a} + (1-0)\cdot \bar{b} = \bar{b}$$

THUS WE CONCLUDE THAT AN INTERVAL BTW POINTS $\bar{a}$ AND $\bar{b}$ CAN BE DESCIBED AS :

$$\{ \lambda \bar{a} + (1-\lambda)\bar{b} \mid \lambda \in [0,1] \}$$

THE INTERVAL BTW $\bar{a}, \bar{b}$ WILL BE DENOTED BY $[\bar{a}, \bar{b}]$

$n=1$ $[1, 3]$

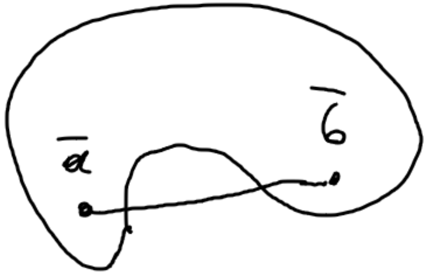


CONVEX SETS

SUBSET $A \subset \mathbb{R}^n$ IS CALLED CONVEX IF FOR ANY $\bar{a}, \bar{b} \in A$ THE INTERVAL $[\bar{a}, \bar{b}] \subset A$

EX

CONVEX



NOT CONVEY

NOT CONVEX
BUT IS
STAR-SHAPED