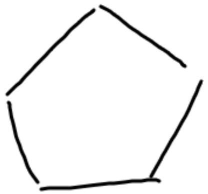
PLATONIC AND ARCHEMEDIAN SOLIDS

PLATONIC – EVERY FACE IS A SAME REGULAR POLYGON

-  $4 \times \triangle$
- 
-  $8 \times \triangle$
- 
- ICOSAHEDRON $12 \times \triangle$

EULER'S FORMULA / CHARACTERISTIC

. FOR ANY SOLID, LET, V - DENOTE THE NUMBER OF VERTICIES, E - DENOTE NUMBUR OF EDGES, F - DENOTE NUMBER OF FACES, THEN

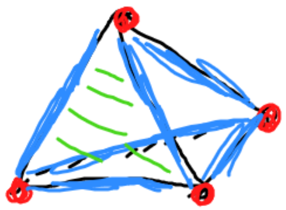
$$V - E + F = 2$$

2 dim SPHERE

$$\begin{array}{c} 2 \\ \parallel \\ \chi(S^2) \end{array}$$

EULER CHARACTERISTIC

Ex



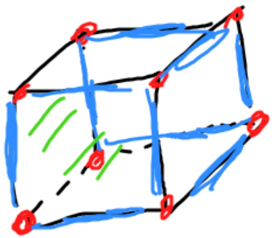
$V = 4$

$E = 6$

$F = 4$

$$4 - 6 + 4 = 2$$

Ex



$V = 8$

$E = 12$

$F = 6$

$$8 - 12 + 6 = 2$$

SKETCH OF AN ARGUMENT FOR CLASSIFICATION OF PLATONIC SOLIDS

$$V - E + F = 2$$

LET'S ASSUME THAT EVERY FACE IS

p - REGULAR POLYGON

$$pF = 2E = qV$$

HOW MANY EDGES MEET AT THE SINGLE VERTEX

$$\frac{2E}{q} - E + \frac{2E}{p} = 2$$

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{2} + \frac{1}{E} \qquad E > 0$$

$$\frac{1}{q} + \frac{1}{p} > \frac{1}{2} \qquad p, q \geq 3$$

$$\{3,3\}, \quad \{4,3\}, \quad \{3,4\},$$

$$\{5,3\}, \quad \{3,5\}$$

THE ONLY PAIRS OF $\{p,q\}$ THAT SATISFIES THIS $\square$

EX $\{3,3\} \longleftrightarrow$ 

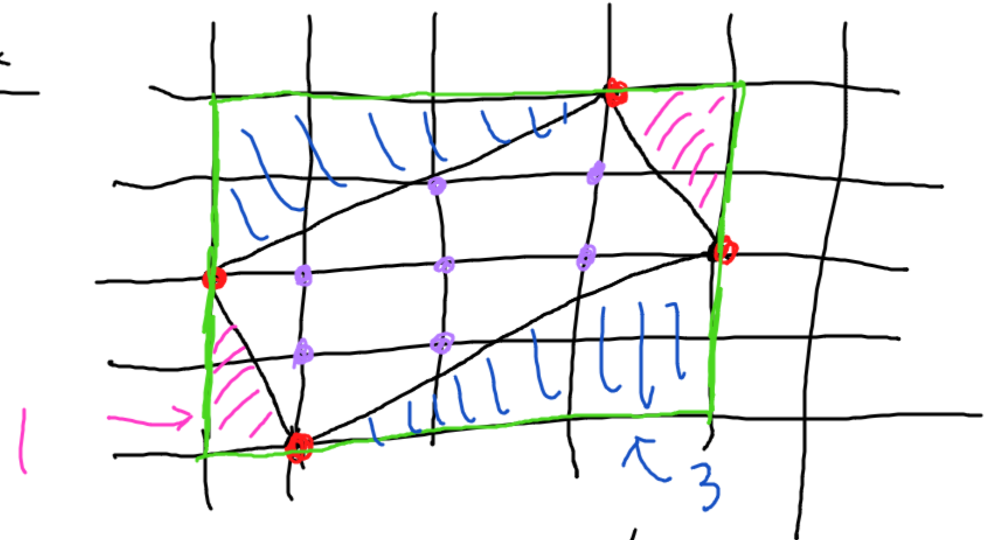
PICK'S TH

- P – POLYGON ON A PLANE, WITH EVERY VERTEX HAVING INTEGER COORDINATES
- i – NUMBERER OF INTEGER POINTS THAT ARE INTERIOR POINTS OF P
- b – NUMBER OF INTEGER POINTS ON THE BOUNDARY OF P.

THEN THE AREA A OF P IS:

$$A = i + \frac{b}{2} - 1$$

Ex



$b = 4$

$i = 7$

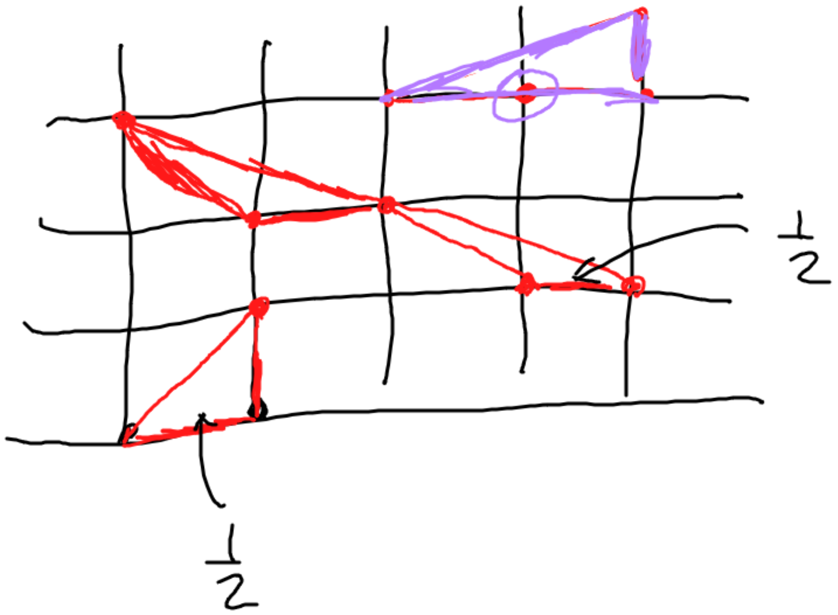
$$A = i + \frac{b}{2} - 1 = 7 + \frac{4}{2} - 1 = 8$$

$$A = 16$$

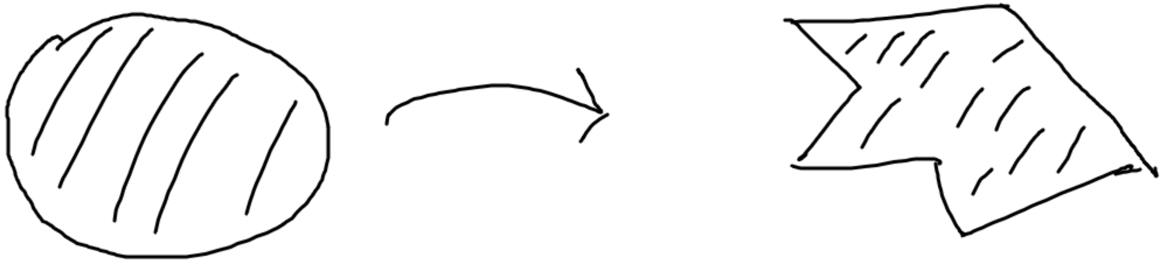
$$A = 16 - 3 - 3 - 1 - 1 = 8$$

SKETCH OF A PROOF:

- LOOK A BASIC TRIANGLES THAT HAVE INTEGER VERTICIES AND NO OTHER INTEGER POINTS



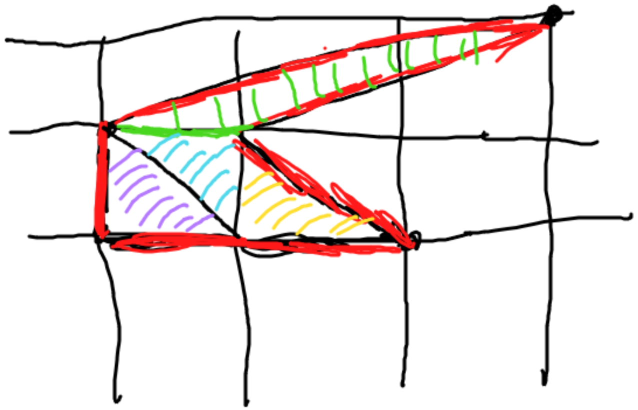
OBSERVE THAT THEY ALL HAVE AN AREA EQUAL TO $\frac{1}{2}$



ANY POLYGON

NOW WE PERFORM A TRIANGULATION OF OUR POLYGON P INTO BASIC TRIANGLES LIKE ABOVE

P



TRIANGULATION OF P INTO A BASIC TRIANGLE

- LET T BE A NUMBER OF TRIANGES

  V BE A NUMBER OF VERTICIES

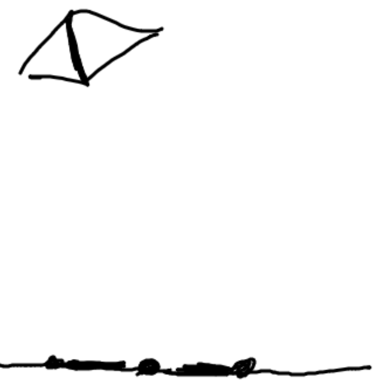
$V = i + 6$

$F = T + 1$

$E = ?$

EVERY $\triangle$ GIVES 3 EDGES

$$E = \frac{3 \cdot T}{2} + \frac{6}{2}$$



• WE USE EULER'S FORMULA

$$2 = (i+b) - \frac{3T+b}{2} + T + 1$$

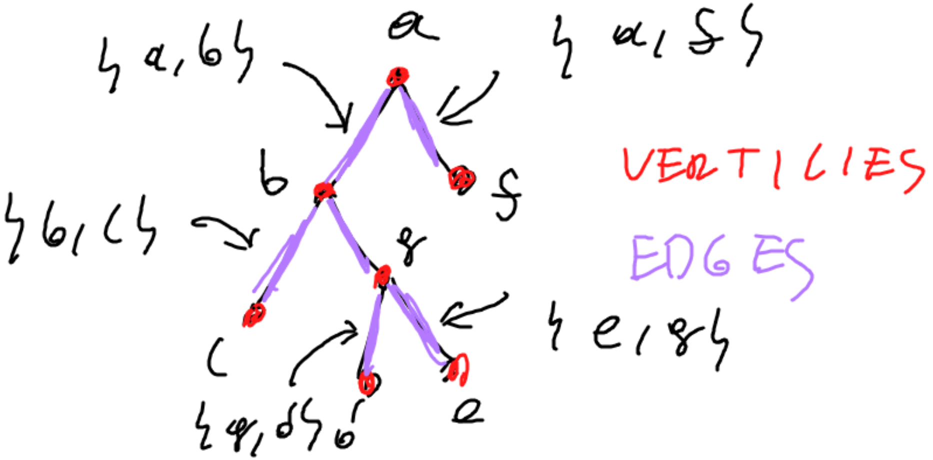
$$T = 2i + b - 2$$

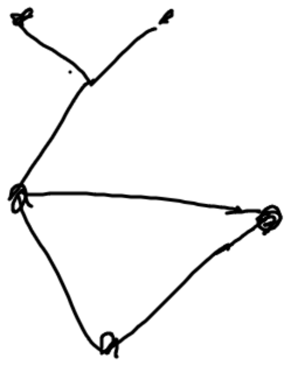
BUT EVERY Δ HAD AREA $\frac{1}{2}$

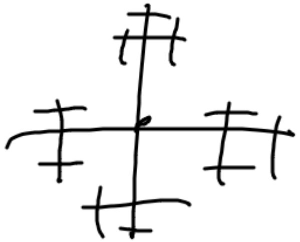
$$A = \frac{T}{2} = i + \frac{b}{2} - 1$$

$\blacksquare$

GRAPH THEORY







$V = \{a, b, c, d, e, f, g\}$

$E = \{ \{a,b\}, \{b,c\}, \{a,f\},$
$\{b,g\}, \{g,d\}, \{g,e\} \}$

"MODELING 1-dim OBJECTS" EX NETWORKS, ROAD SYSTEMS, PROPAGATION OF INF

DEF GRAP $G = (V, E)$ IS A PAIR OF SETS V - SET OF VERTICIES, AND $E \subset V \times V$ - SET OF EDGES.

- $V = \{a, b, c, d\}$

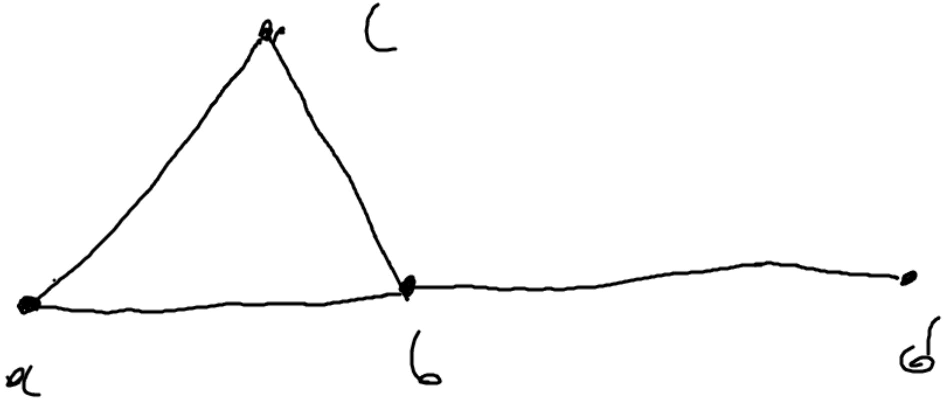
$E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}$

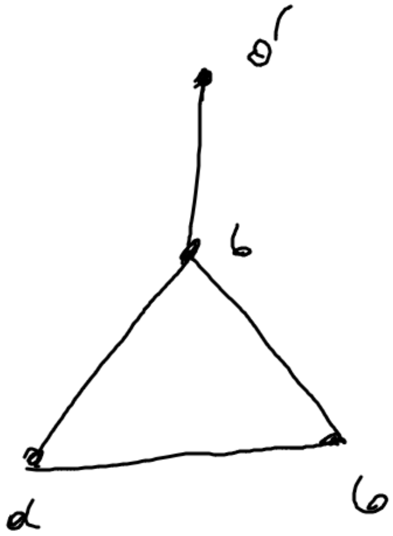
$\cap$

$V \times V$ i.e AN ELEMENT OF E CONSITS OF A COUPLE OF ELEMENTS OF V

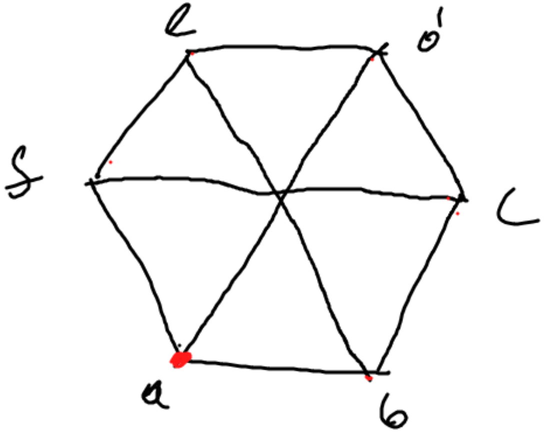
CARTISIAN PRODUCT

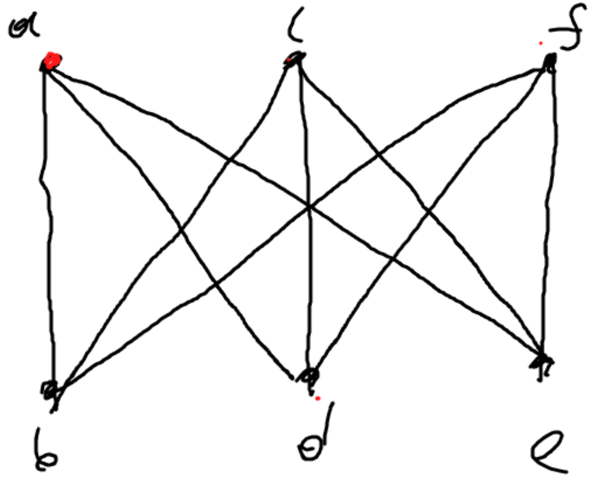
$\mathbb{R}, \quad \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}, \quad \mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$





NOTE THAT
THIS GRAPHICAL
REPRESENTATION
/ GEOMETRIC
REP.
IS <u>NOT</u> UNIQUE





BOTH ARE GEOMETRIC RED

$V = \{ a, b, c, d, e, f \}$

$E = \{ \{a, b\}, \{a, f\}, \{a, d\},$
$\{b, c\}, \{d, c\}, \{c, e\},$
$\{f, b\}, \{f, d\}, \{f, e\} \}$

<u>DEF</u> WE'RE GOING TO SAY THAT TWO GRAPHS $G = (V, E)$ AND $H = (V', E')$ ARE ISOMORPHIC (SAME FROM GRAPH-THEORETIC POINT OF VIEW) IF WE CAN ASSIGN ALL ELEMENTS OF V TO V' IN SUCH THAT RESPECTS EXISTENCE OF EDGES i.e.

- $f : V \to V'$ BIJECTION i.e. $f(a) = f(b) \Rightarrow a = b$
- $\{a, b\} \in E \iff \{f(a), f(b)\} \in E'$