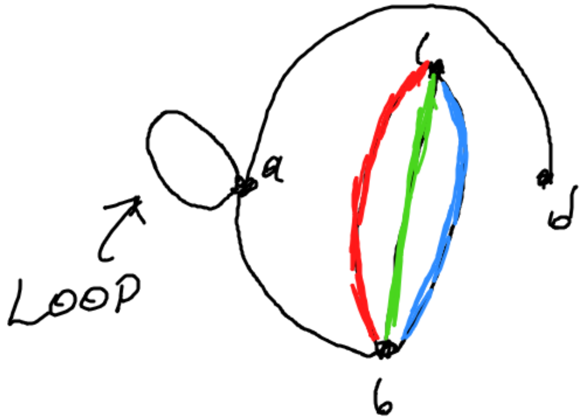
RECALL

GRAPHS $G = (V, E)$

V — VERTICIES, E — EDGES



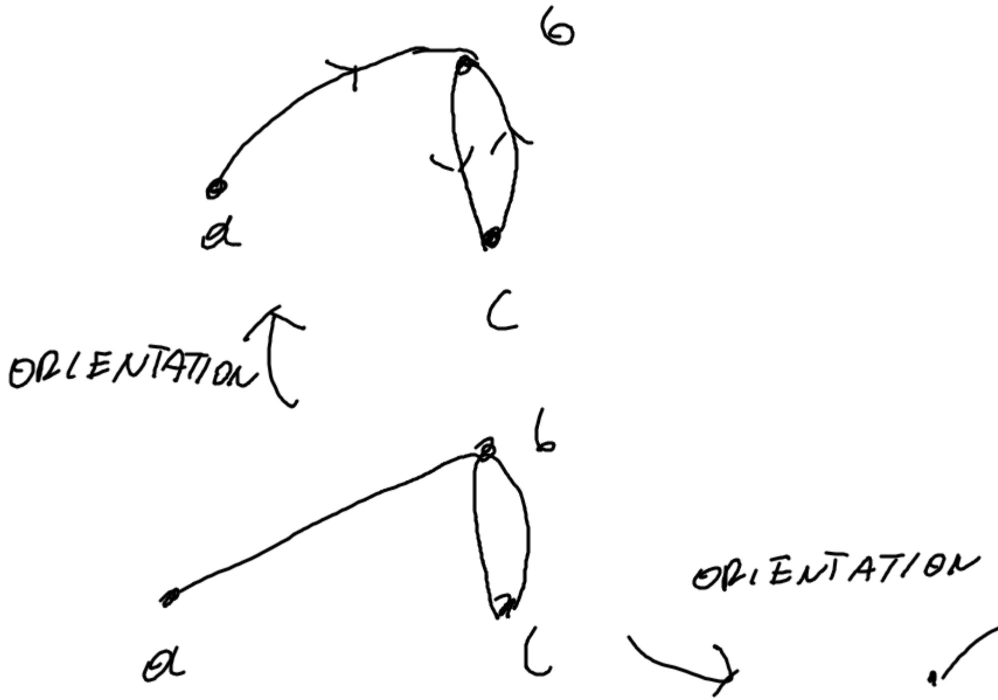
DEF SIMPLE GRAPHS ARE GRAPHS WITHOUT LOOPS OR MULTIPLE EDGES FOR FIXED PAIR OF VERTICIES

$\{a, a\} \subset E$
LOOP WHICH IS NOT ALLOWED IN SIMPLE GRAPHS

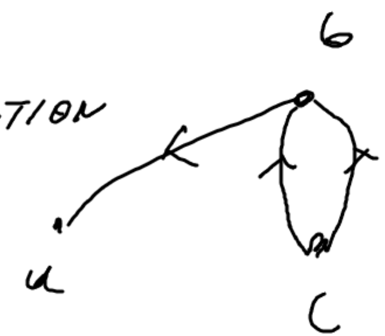
WE HAVE 3 EDGES BTW b AND c :

NOTE : THAT FOR A SIMPLE GRAPH AN EDGE IS UNIQUELY DEFINED BY VERTICIES IT CONNECTS

RMK WE CAN ALSO CONSIDER DIRECTED GRAPHS WHERE EDGES HAVE ORIETATION



EVERY UNORIENTED GRAPH CAN BE MADE ORIENTED SIMPLY BY PICKING AN ORIENTATION FOR EVERY EDGE. NOTE THAT THIS CHOICE IS **NOT** UNIQUE.

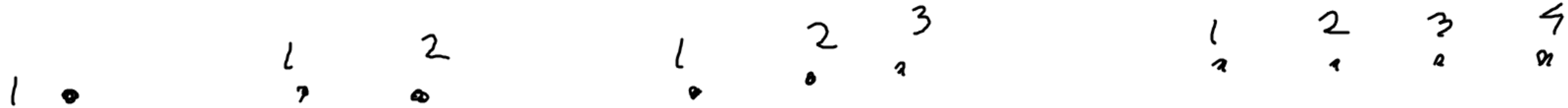


EXAMPLES OF GRAPHS (SIMPLE)

- $A_n = \{\{1, \dots, n\}, \varnothing\}$ EMPTY GRAPH

  $|E| = 0$

  ↑ EMPTY SET



A_1 A_2 A_3 A_4

- K_n - FULL GRAPH $V(K_n) = \{1 \dots n\}$

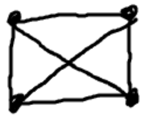
  THERE IS AN EDGE BTW EVERY TWO VERTICES

$|E(K_n)| = \frac{n(n-1)}{2}$

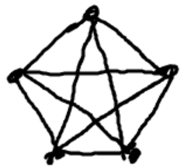
K_1

K_2

K_3

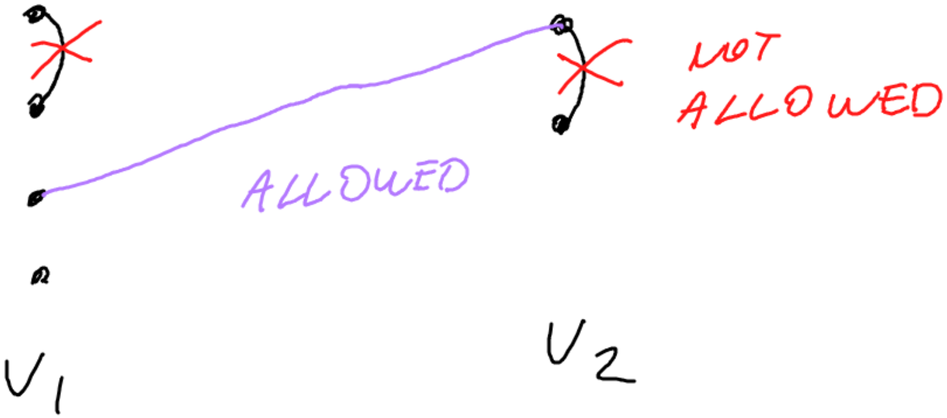


K_4



K_5 ...

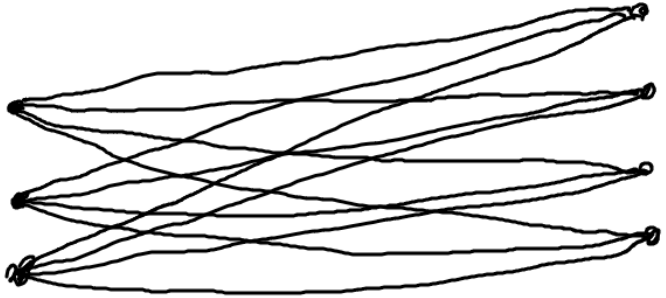
BIPARTITE GRAPH IS A GRAPH WHERE SET OF VERTICLES V CAN BE WRITTEN AS A SUM OF TWO SETS V_1 AND V_2 $V = V_1 \cup V_2$ SUCH THAT ANY TWO VERTICIES THAT BELONG TO V_i ARE NOT CONNECTED BY AN EDGE



• FULL BIPARTITE GRAPHS $K_{n,m}$

$|V_1| = n$ $|V_2| = m$ AND THERE IS AN EDGE BTW EVERY $v \in V_1$ AND $w \in V_2$

$K_{3,4}$

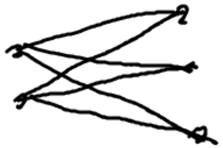


$|V_1| = 3$ $|V_2| = 4$

$|E(K_{n,m})| = n \cdot m$

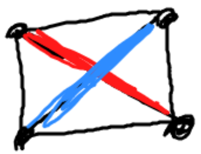
$K_{1,2}$

$K_{2,3}$



- PLANNAR GRAPH : IS A GRAPH THAT CAN BE DRAWN ON A PLANE $\mathbb{R}^2$ WITHOUT EDGES INTERSECTING

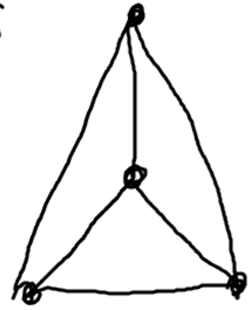
THIS CAN BE HARD TO DECIDE BC THE SAME GRAPH CAN BE DRAWN IN DIFFERENT WAYS



K 4

IN THIS REP. | AND | INTERSECT SO THIS IS NOT PLANAR

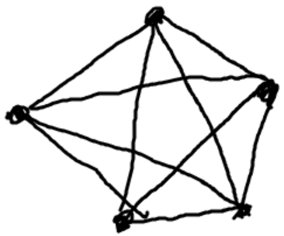
BUT, HERE'S ANOTHER REP OF THE SAME GRAPH, THAT IS PLANAR



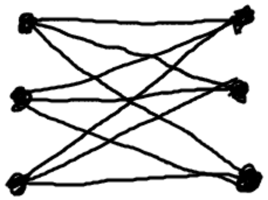
K 4

TH (KURATOWSKI 1930)

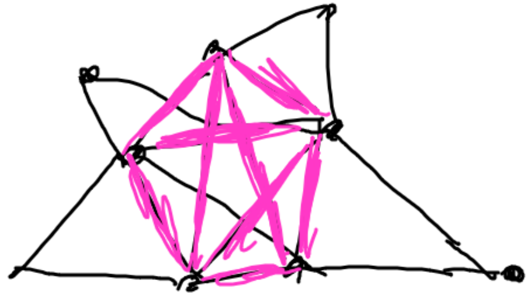
GRAPH IS PLANAR IF AND ONLY IF IT DOES NOT CONTAIN A SUBGRAPH ISOMORPHIC TO K_5 OR $K_{3,3}$.



K_5

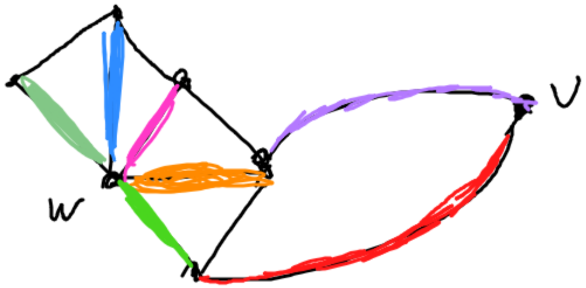


$K_{3,3}$



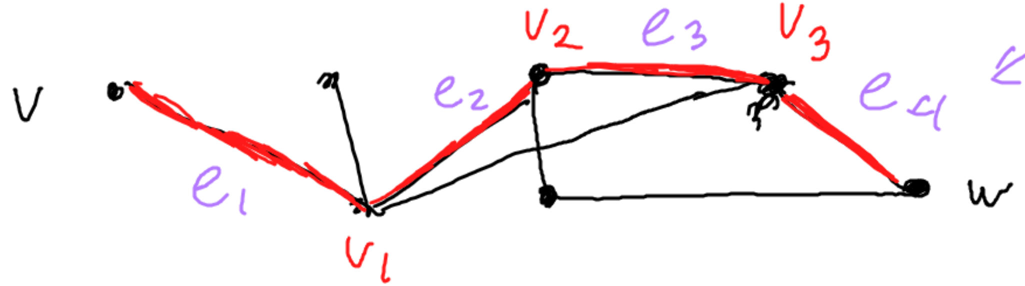
IT CONTAINS K_5 SO IT'S PLANAR

- $\deg(v)$ ⟵ DEGREE OF A VERTEX IS A NUMBER OF EDGES THAT ARE CONNECTED TO v



$\deg(v) = 2$
$\deg(w) = 5$

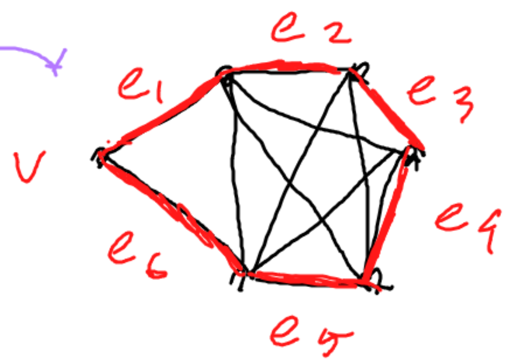
- PATH BTW A VERTEX v AND A VERTEX w IS A SEQUENCE OF EDGES $\{v\ v_1\} \to \{v_1\ v_2\} \to \{v_2, v_3\} \to ,,, \{v_n \to w\}$. AND $n+1$ WHICH IS A NUMBER OF EDGES IS CALLED THE LENGTH OF THIS PATH



SIMPLE PATH

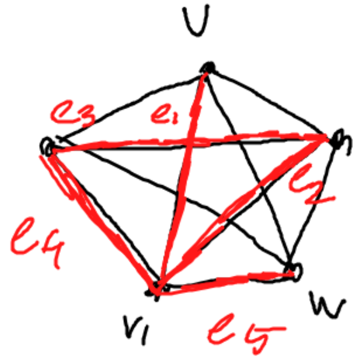
- CYCLE IS A PATH WHERE THE BEGGINING IS THE SAME AS THE END $V = W$

SIMPLE CYCLE

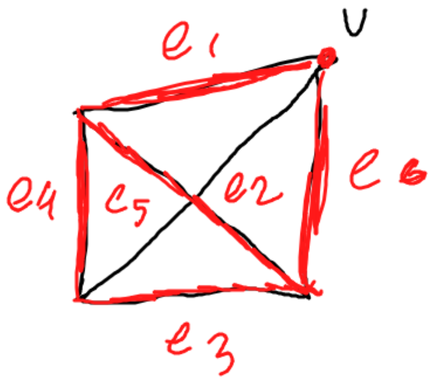


CICLE i.e. A PATH THAT STARTS AND ENDS AT V. AND ITS LENGTH IS EQUAL TO 6.

- PATH / CYCLE IS SIMPLE IF IT DOESN'T VISIT ANY VERTEX MORE THAN ONCE



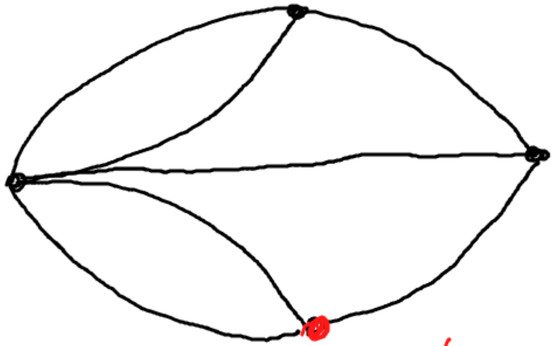
PATH BTW V AND W, WHICH IS NOT SIMPLE.



CICLE STARTING AND ENDING AT V, WHICH IS NOT SIMPLE

EULER GRAPHS

KONIGSBERG BRIDGES



NOT EULER

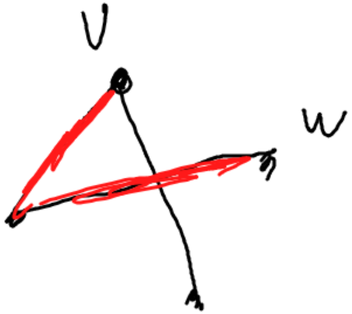
deg = 3

CAN YOU GO THROUG EVERY BRIDGE aka EDGE AND RETURN TO THE SAME SPOT AS YOU STARTED

DEF EULER CYCLE IS A CYCLE THAT VISITS EVERY EDGE, EXACTLY ONCE

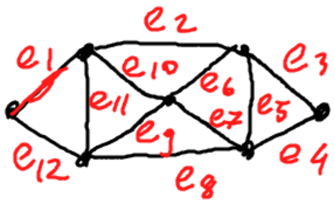
GRAPH IS EULER IF IT HAS AN EULER CYCLE

TH A GRAPH IS EULER IF AND IF IT'S CONNECTED (YOU CAN FIND A PATH BTW ANY TWO VERTICIES) AND A DEG OF EVERY VERTEX IS EVEN



IS CONNECTED

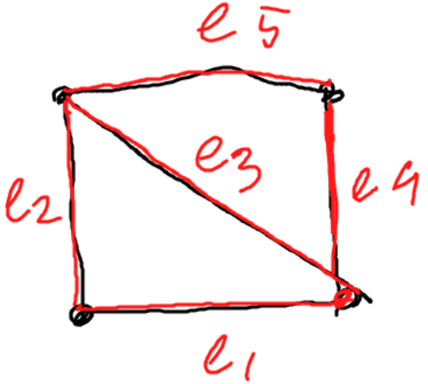
NOT CONNECTE: NO PATH FROM V TO W.



← EULER GRAPH

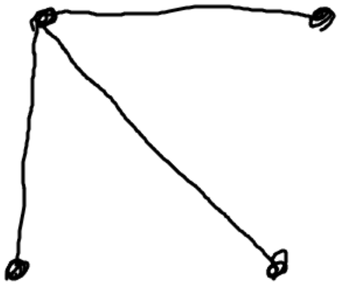
HAMILTONIAN GRAPHS

DEF A CYCLE IS HAMILTONIAN IF IT GOES THROUGH EVERY VERTEX EXACTLY ONCE.



HAMILTONIAN

NOT HAMILTONIAN



DEF GRAPH IS HAMILTONIAN IF IT HAS A HAMILTONIAN CYCLE

IT TURNS OUT THAT CLASSIFICATION OF HAMILTONIAN GRAPHS IS REALLY HARD

THIS HAS A LOT APPLICATIONS, CALLED A SALESMAN PROBLEM.

NOT ONLY NO CLASSIFICATION SO FAR, BUT ALSO THE PROBLEM IS NP-HARD.

SOME PARTIAL RESULTS, FOR EXAMPLE:

TH (ORE 1960)

IF A SIMPLE GRAPH WITH AT LEAST 3 VERTIC. SATISFIES THAT FOR ANY $v, w \in V$

$\deg(v) + \deg(w) \geq |V|$ THEN IT'S HAMILTONIAN