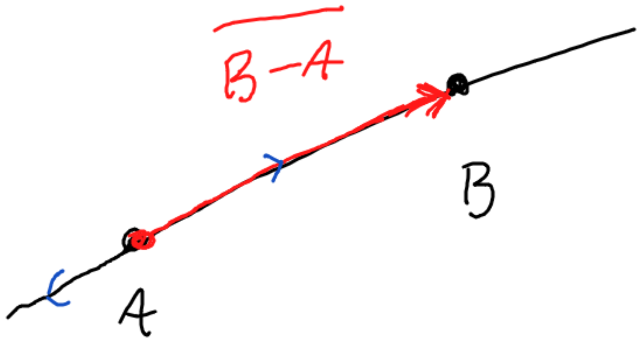
CURVES

"LINE THAT DOESN'T NEED TO BE STRAIGHT"



LINE

- EVERY PAIR OF POINTS DEFINSES A LINE GOING THROUGH THEM
- USING AN EQUATION

  $y = ax + b$

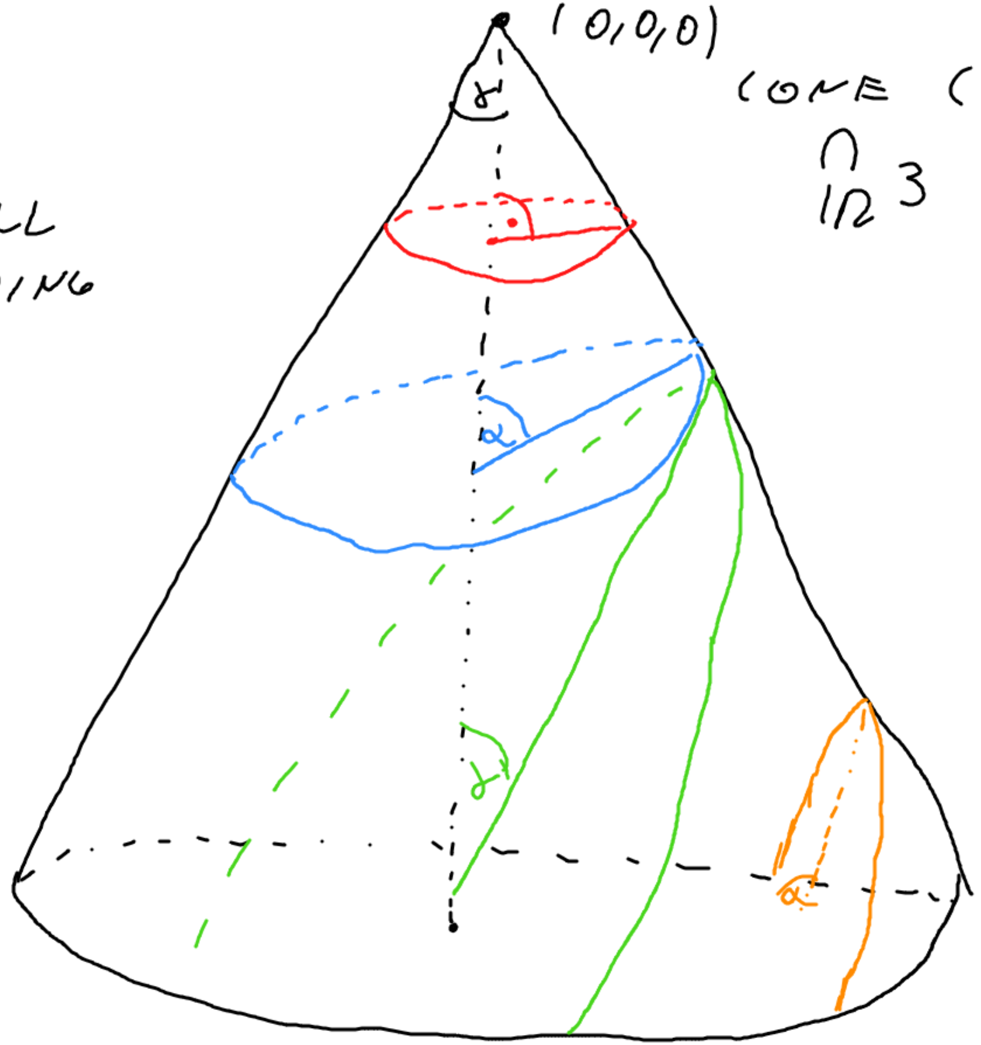
  $a, b \in \mathbb{R}$ PARAMETERS
- $A = (x_0, y_0)$

  $B - A = (v_x, v_y)$

PARAMETRISATION $\longrightarrow$ $\{(x_0 + t v_x, y_0 + t v_y) : t \in \mathbb{R}\}$

CONIC SECTIONS

- IF WE INTERSECT A CONE C WITH DIFFERENT PLANES WE'LL GET DIFERENT CURVES DEPENDING ON THE ANGLE α OF THOSE PLANES ARE CONE'S AXIS
- CIRCLE $\alpha = \frac{\pi}{2}$
- ELLIPSE $\gamma < \alpha < \frac{\pi}{2}$
- PARABOLA $\gamma = \alpha$
- HYPERBOLA $0 < \alpha < \gamma$



CONE $C \cap \mathbb{R}^3$

CIRCLES

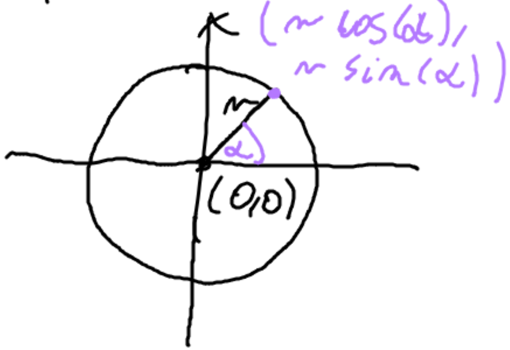
- A SET OF POINTS THAT IS A FIXED DISTANCE r APART FROM ITS CENTER (x_0, y_0)

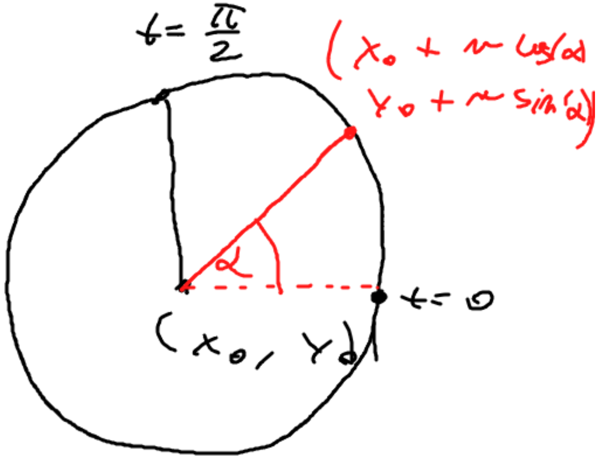
- $r^2 = (x - x_0)^2 + (y - y_0)^2$ ← EQUATION

- PARAMETRISATION

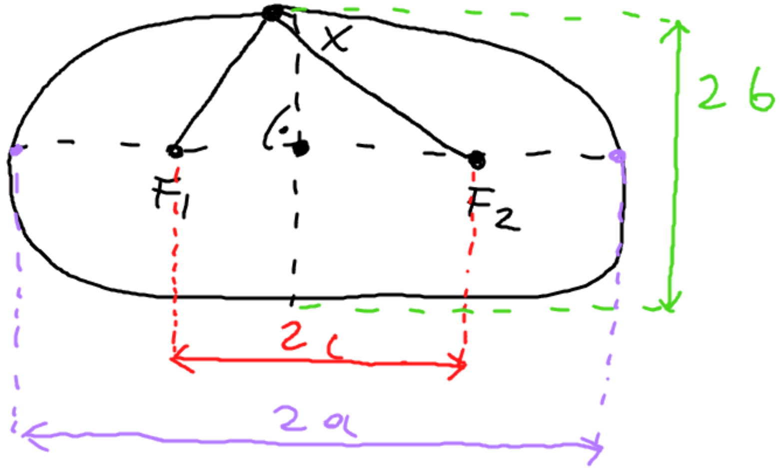
$$(x_0 + r\cos(t),\ r\sin(t) + y_0)$$

$t \in \mathbb{R}$





ELLIPSE



- SET OF POINTS x ON A PLANE s.t. SUM OF DISTANCES TO A PAIR OF FOCAL POINTS IS FIXED

$$d(x, F_1) + d(x, F_2) = \text{const}$$

"ELIPSE WITH e CLOSER TO 1 IS CLOSER TO A CIRCLE IN SHAPE"

$e = \frac{c}{a} < 1$ DIRECTIX

$e = 1$ IF ELIPSE IS A CIRCLE i.e $F_1 = F_2$

2a – LENGTH OF MAYOR AXIS

2b – LENGTH OF MINOR AXIS

- ASSUME $a > b$ FOCI ARE $(\pm c, 0)$

$$\frac{x^2}{a} + \frac{y^2}{b} = 1 \leftarrow \text{EQUATION}$$

$$(a \cos(t), b \sin(t)) \leftarrow \text{PARAMETRISATION}$$

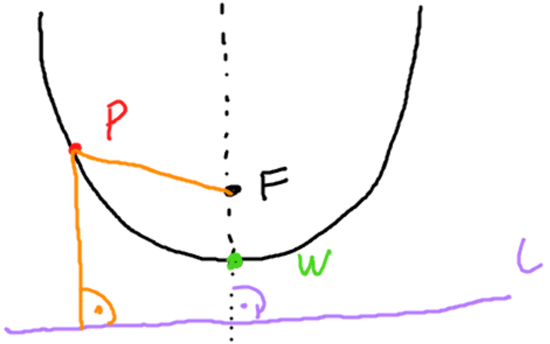
- IF THE CENTER OF THE ELLIPSE IS AT (x_0, y_0):

$$\frac{(x-x_0)^2}{a} + \frac{(y-y_0)^2}{b} = 1 \Leftarrow \text{EQUATION}$$

$$\big(x_0 + a\cos(t),\ y_0 + b\sin(t)\big) \Leftarrow \text{PARAMETRISATION}$$

PARABOLA

- SET OF POINTS s.t. THE DISTANCE TO A FIXED POINT F (FOCUS) AND A FIXED LINE l (DIRECTIX) IS THE SAME



$d(P,F) = d(P,l)$

W - VERTEX OF A PARABOLA

$W = (x_0, y_0)$

- EQUATIONS IF THE AXIS OF SYMMETRY IS $\parallel$ OY, THEN

$$(x - x_0)^2 = 2p(y - y_0)^2$$

P - PARAMETER

THIS CAN BE RE-WRITTEN AS

$$y = ax + bx + c \qquad a \neq 0$$

$$x_0 = -\frac{b}{2a}, \quad y_0 = -\frac{\Delta}{4a}$$

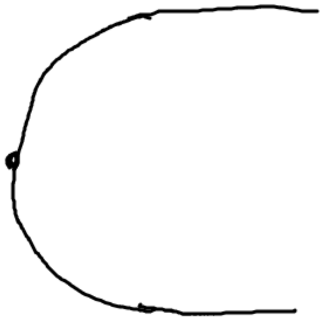
$$\Delta = b^2 - 4ac$$

"PARABOLAS ARE GRAPHS OF A QUADRATIC EQ."

$(2at, at^2) \leftarrow$ PARAMETRISATION

• IF THE AXIS OF SYMMETRY IS $\parallel$ OX

$W = (x_0, y_0)$

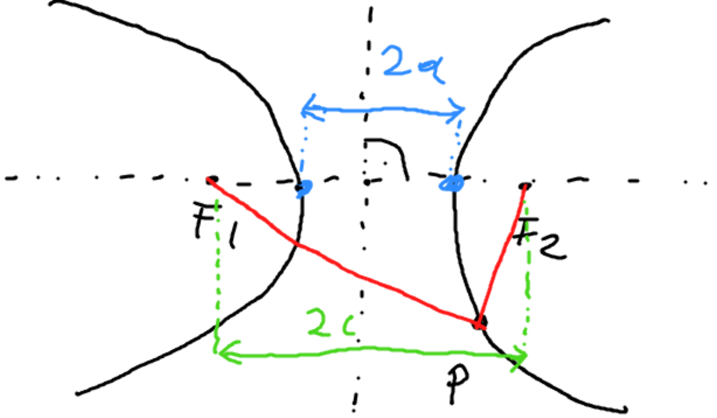


$$(y - y_0)^2 = 2p(x - x_0)^2$$

EQUATION

$$(at^2, 2at)$$

HYPERBOLA



- SET OF POINTS s.t. ABSOLUTE VALUE OF A DIFFERENCE OF DISTANCES TO A PAIR OF FIXED POINTS F_1, F_2 IS FIXED.

$$| d(P, F_1) - d(P, F_2) | = \text{const}$$

$$(= 2a) \leftarrow$$ CONSIDER ONE OF THE VERTICES

- IF HYPERBOLA IS CENTERED AT $(0, 0)$

EQUATION $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\qquad b = \sqrt{c^2 - a^2}$

PARAMETRISATION $(a \cosh(t), b \sinh(t))$

$\cosh$: HIPERBOLIC COS

$\sinh$: HIPERBOLIC SIN

$$\cosh^2(t) - \sinh^2(t) = 1$$ $\leftarrow$ "TRIGONOMETRIC" IDENTITY

- IF THE HYPERBOLA IS CENTERED AT (x_0, y_0):

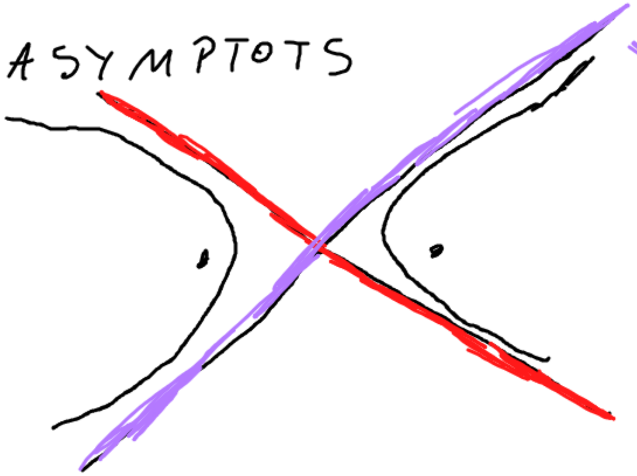
$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1 \qquad b = \sqrt{c^2 - a^2}$$

EQUATION

PARAMETRISATION

$$\left(x_0 + \cosh(t),\ y_0 + \sinh(t) \right)$$

- ASYMPTOTS



EQUATION OF ASYMPTOTS:

$$y - y_0 = -\frac{b}{a}(x - x_0)$$

$$y - y_0 = \frac{b}{a}(x - x_0)$$