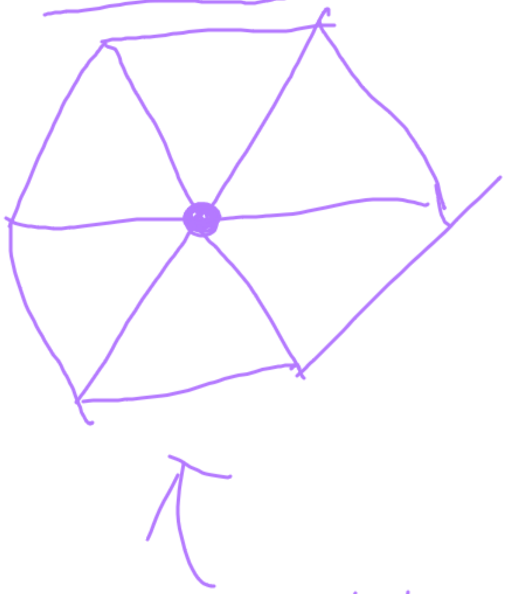
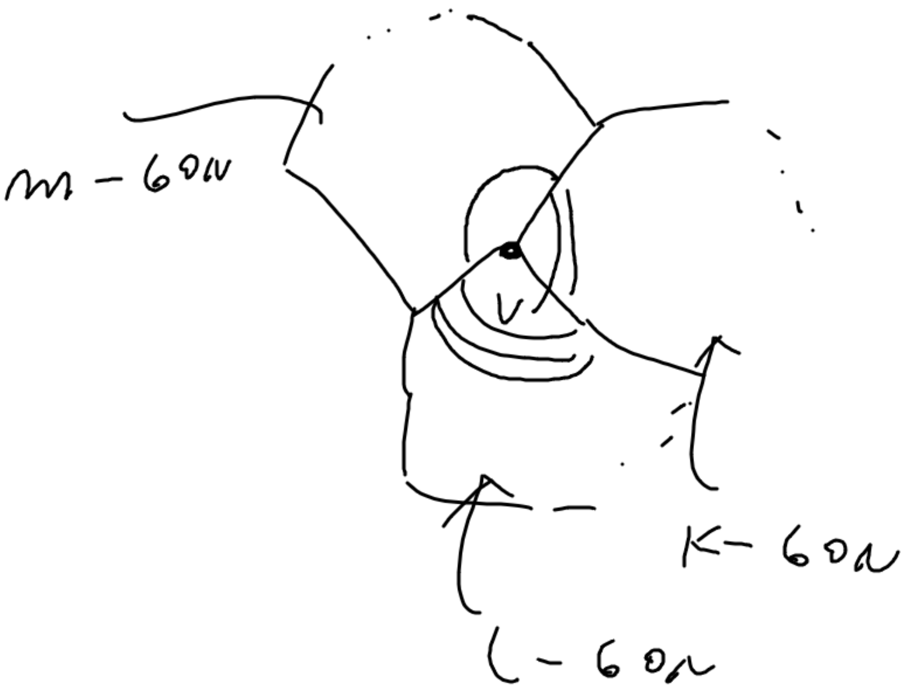
REGULAR



REGULAR

AT EVERY
VERTEX 6
TRIANGLES
MEET

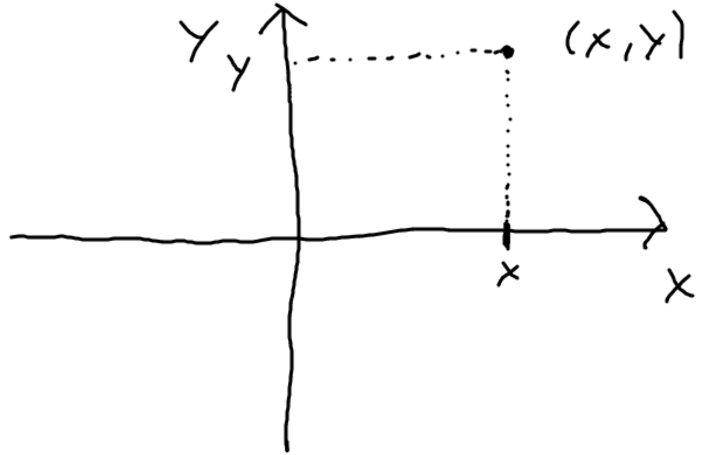
SEMI-REGULAR



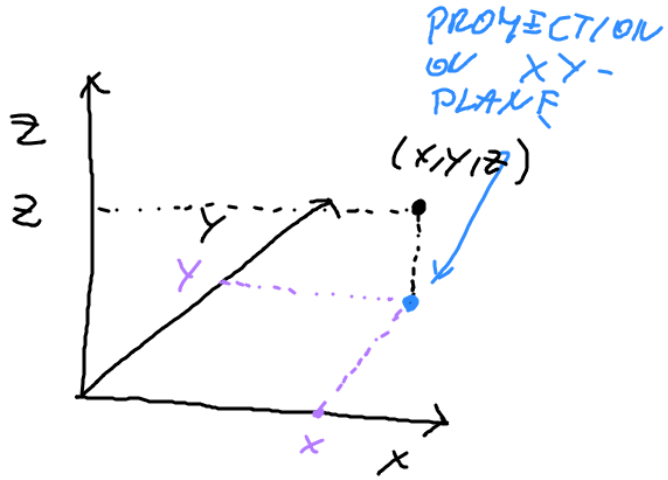
SURFACES ~ CURVED PLANES

BEFORE THAT LET'S TALK ABOUT DIFFERENT COORDINATE SYSTEMS

CARTISIAN COORDINATES



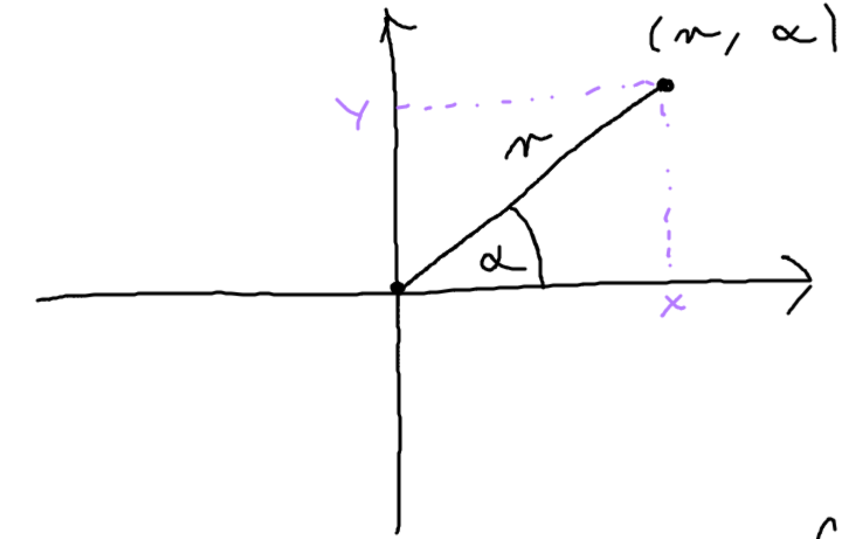
2 - dim



3 - dim

WE CAN KEEP DOING THIS IN HIGHER dim

POLAR COORDINATES



r – DISTANCE TO (0,0)

α – THE ANGLE BTW OX AXIS AND THE SEGMENT FROM POINT TO (0,0)

$$\begin{cases} \cos(\alpha) = \frac{x}{r} \\ \sin(\alpha) = \frac{y}{r} \end{cases} \leadsto \begin{cases} x = r \cdot \cos(\alpha) \\ y = r \cdot \sin(\alpha) \end{cases} \leadsto x^2 + y^2 = r^2(\sin^2\alpha + \cos^2\alpha) = r^2$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan(\alpha) = \frac{y}{x} \end{cases}$$

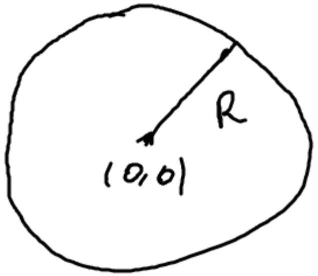
$x > 0$

$$\Downarrow$$

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

SOME CURVES WILL BE EASIER TO DISCRIBE USING POLAR COORDINATES

EX



POLAR COORDINA

$r = R$

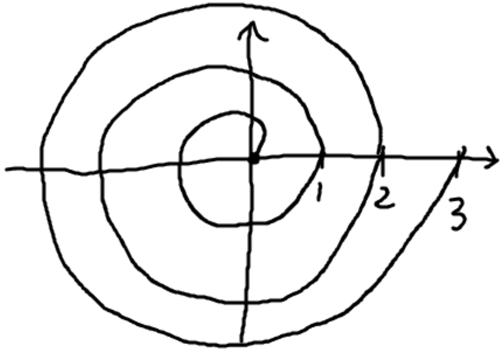
$\alpha \in (0, 2\pi)$

CARTISIAN COORD.

$x^2 + y^2 = R^2$

EQUATION

EX



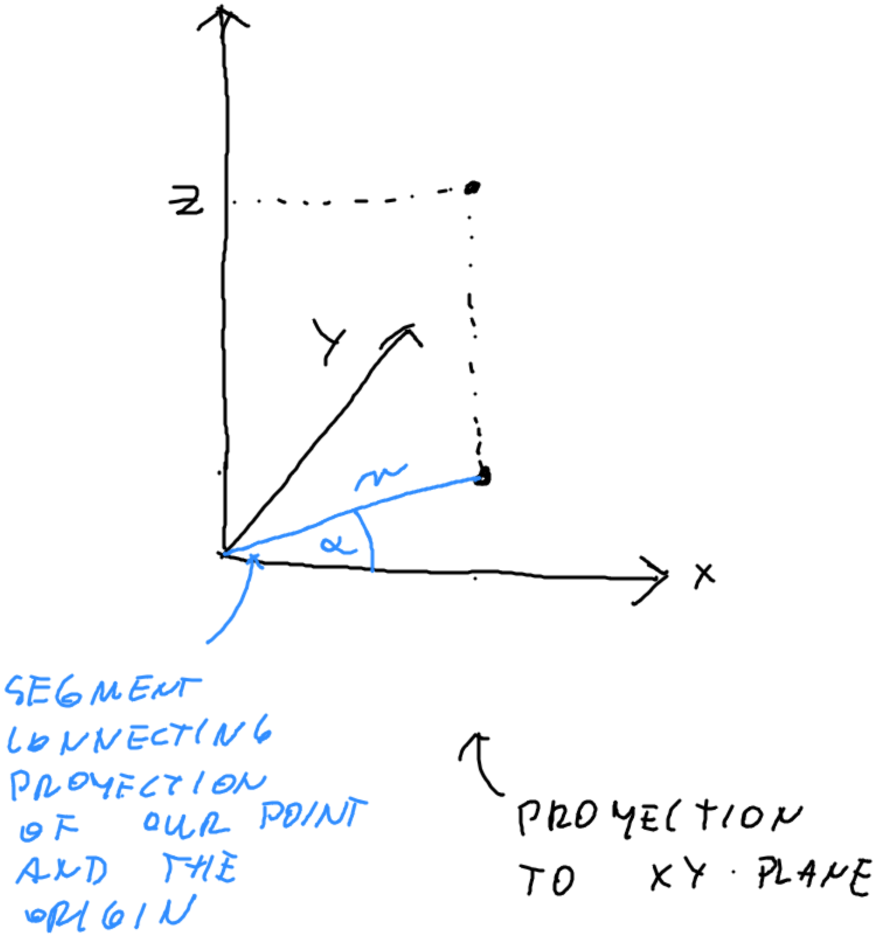
$r = t$

$\alpha = 2\pi \cdot t$

???

PARAMETRISATION

CYLINDRICAL COORDINATES



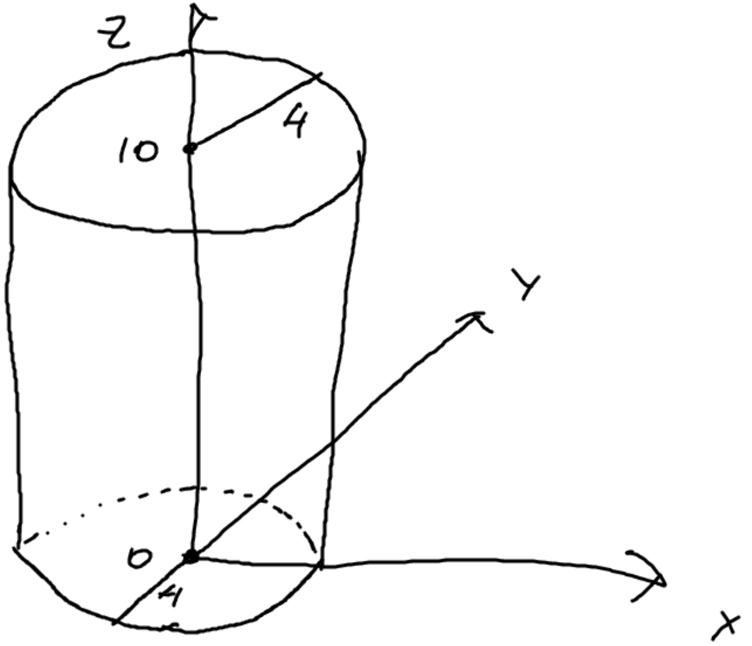
$$\begin{cases} x = r\cos(\alpha) \\ y = r\sin(\alpha) \\ z = z \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan(\alpha) = \dfrac{y}{x} \end{cases}$$

CYLINDERS:

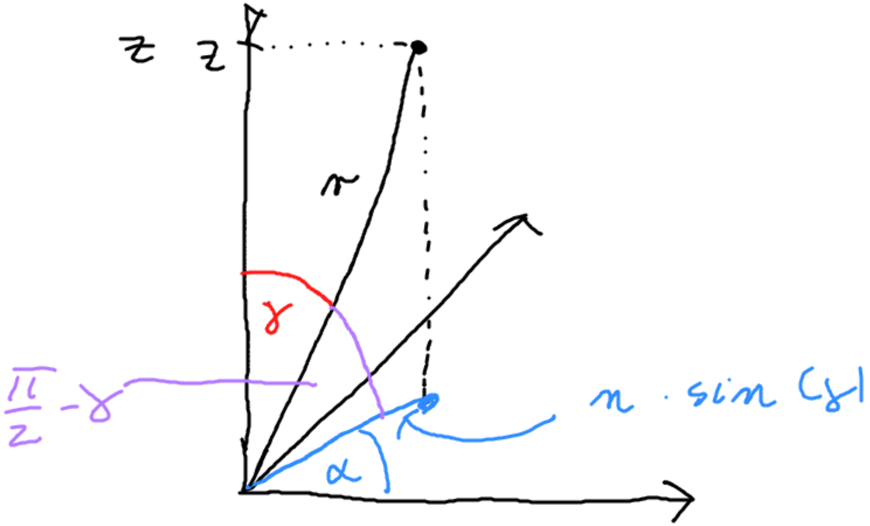
$$\begin{cases} r = 4 \\ \alpha \in [0, 2\pi] \\ z \in [0, 10] \end{cases}$$



$$x^2 + y^2 = r^2$$

EQUATION IN CARTESIAN COORD.

SPHERICAL COORDINATES



• PROJECTION TO XY PLANE

$r \geq 0$

$\alpha \in [0, 2\pi]$

$\gamma \in [0, \pi]$

r - THE DISTANCE FROM THE ORIGIN TO OUR POINT

α - THE ANGLE BTW OX - AXIS AND THE SEGMENT FROM THE ORIGIN TO THE PROJECTION OF THE POINT

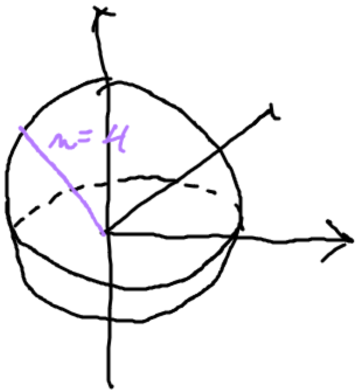
γ - THE ANGLE BTW OZ AXIS AND THE LINE BTW THE ORIGIN AND THE POINT

$$\begin{cases} x = r \cos(\alpha) \sin(\gamma) \\ y = r \sin(\alpha) \sin(\gamma) \\ z = r \cos(\gamma) \end{cases} \iff \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \gamma = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) \\ \tan(\alpha) = \dfrac{y}{x} \end{cases}$$

EX

SPHERE

$r = 4 \qquad \alpha \in (0, 2\pi) \qquad \gamma \in (0, \pi)$

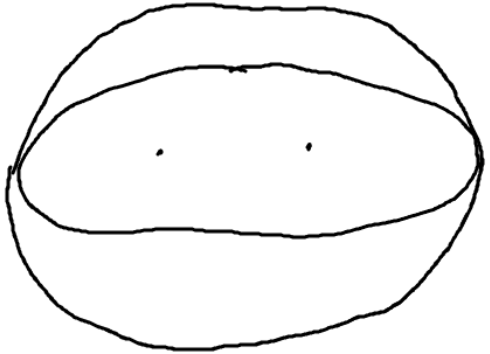


$$x^2 + y^2 + z^2 = R^2$$

QUADRATIC SURFACES

(ANALOG OF CONIC CURVES FOR SURFACES)

- ELIPSOID



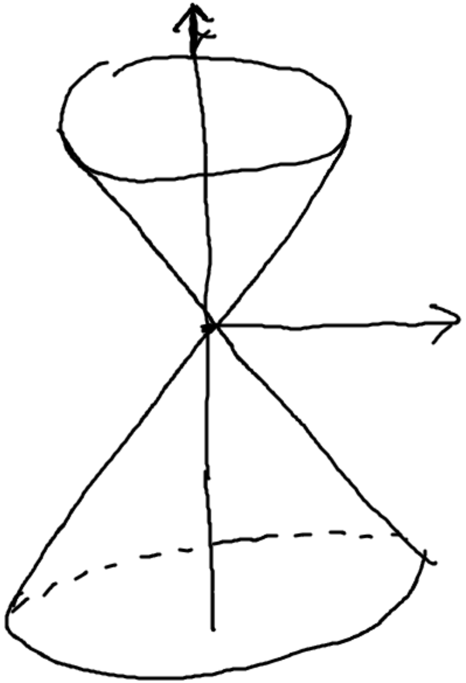
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$a = b = c$ SPHERE

$$\frac{x^2 + y^2 + c^2}{a^2} = 1$$

$$x^2 + x^2 + c^2 = a^2$$

- CONES



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

PARAMETRISATION IN CYLINDRICAL COORDINATES

$z \in \mathbb{R}$ - ANY NUMBER

$\alpha \in (0, 2\pi)$ - ANY ANGLE

$r = \alpha \cdot t$

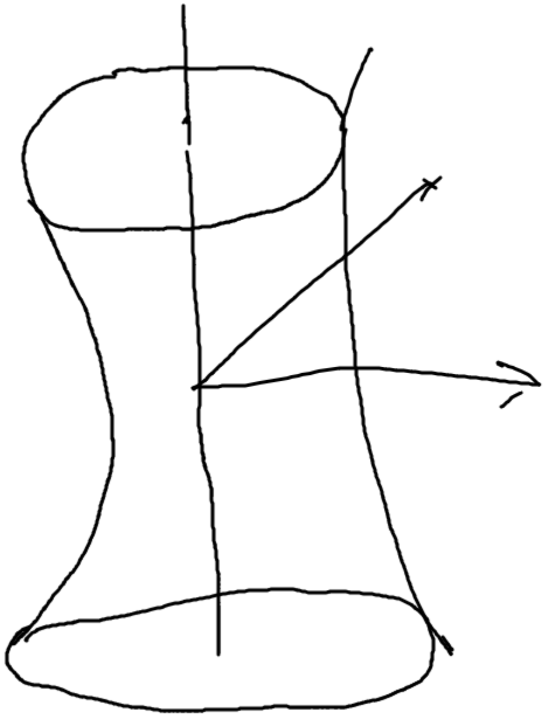
α ← CONST

t ← PARAMETER

- CYLINDERS (SEE EARLIER) $x^2 + y^2 = R^2$

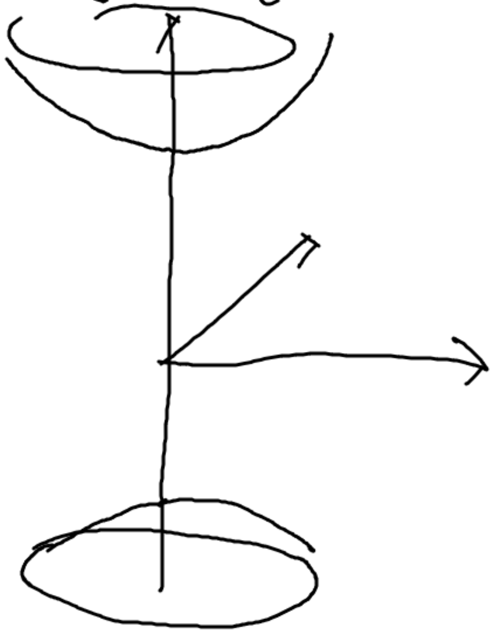
HYPERBOLOID

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



1 - SHEET

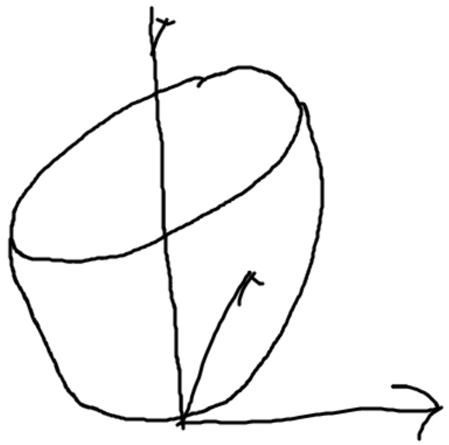
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$



2 - SHEET

ELLIPTIC PARABOLOID

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$



INTERSECTIONS WITH PLANES ∥ XY PLANE ARE ELIPSES (OR CIRCLES IF $a = b$)

INTERSESTIONS WITH PLANE ⊥ XY-PLANE ARE PARABOLAS

HYPERBOLOIC PARABOLOID

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$

- Z - FIXED

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{const}$$

- X OR Y - FIXED

WE GET PARABOLAS BUT ONE FACING UP AND THE OTHER FACING DOWN

THIS IS A FULL LIST OF QUADRATIC SURFACES, SO SURFACES WHICH ARE GRAPHS OF QUADRATIC EQUATIONS IN 3-dim:

$$0 = Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hx + Iz + J$$

$A, B, \ldots, J$ - CONST