

Game Theory and Applications
Problem set 7

1. Find the Nash bargaining solution for bargaining set

$$W = \left\{ (u, v) : \frac{u}{2} \leq v \leq \frac{1}{2} - u^2 \right\}$$

and disagreement point $(u^*, v^*) = (-1, -\frac{1}{2})$.

2. Find the Nash bargaining solution for a bimatrix game defined by matrices:

(a)

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 1 \\ 6 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}.$$

(b)

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 & 1 \\ -1 & -1 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$

Assume that the disagreement point corresponds to security levels of the players.

3. Consider the following cooperative game: Player 1 wants to sell his house. He estimates its value at 1 mln Euros. Each of Players 2 and 3 is interested in buying it. Both estimate its value at 2 mln. and both have such sum at their disposal. Value of a coalition is computed as follows: if player 1 and at least one of the other players are in the coalition, the house is sold, and so the value of the coalition is the sum of the money that the coalition members have plus the value of the house for the buyer. Similarly, for a coalition which cannot close any deal the value is the sum of money its members possess (for coalitions without Player 1) or the value of the house for Player 1 if he is the only person in the coalition. Define the characteristic function for this game. Find its core.
4. Let v be a weighted voting game with weights $w_1 = 6, w_2 = 1, w_3 = 2, w_4 = 4$ Compute the Shapley value for this game. Show that its core is empty.
5. A double majority is needed to make a decision in some international organization (that is, most of the countries with majority of inhabitants need to vote for it). Compute the Shapley value for this organization assuming, that the organization consists of 5 countries: A with 80 mln. inhabitants, B with 50 mln., C with 30 mln., D with 20 mln. and E with 10 mln.
6. Christian Unity Union is a federation of four parties: Christian Democrats (10 members), Christian Monarchists (4 members), Christian Communists (4 members) and Christian Anarchists (2 members). Apart from them, there is one more member of the Union – Honorary President – who is not a member of any of these parties. All the decisions in the Union are made by the Executive Board consisting of one member of each party and the Honorary President in the following way: each decision has to be supported by at least half of the parties and the President or by most of the members of the Union (we assume that each member of a party has the same point of view as its representative).

Define the characteristic function for this game. Compute the Shapley value.

7. Let v be a cooperative game defined as follows: $v(\emptyset) = 0, v(\{a\}) = v(\{b\}) = 1, v(\{c\}) = v(\{a, b\}) = 2, v(\{a, c\}) = v(\{b, c\}) = 4, v(\{a, b, c\}) = 5$.
- (a) Find its core.
- (b) Compute the Shapley value for this game.
- (c) Is Shapley value inside the core?