Title of my talk:

Space and time inversions of stochastic processes and Kelvin transform

Abstract

For a Brownian motion with values in $\mathbb{R}^d$ or, more generally, for a rotationally invariant α-stable process (X_t) with values in $\mathbb{R}^d$

$$(*) \qquad (I(X_{\gamma_t}),\, t \geqslant 0) \stackrel{(d)}{=} (X_t^h,\ t \geqslant 0),$$

where $I(x) = x/\|x\|^2$ is a spherical inversion of $\mathbb{R}^d \setminus \{0\}$, X^h is a Doob h-transform of X with $h(x) = \|x\|^{\alpha-d}$ and γ_t is the inverse of the additive functional $A_t = \int_0^t \|X_s\|^{-2\alpha}\, ds$. We say that a process has "a space inversion property" if it fulfills condition $(*)$.

On the other hand, if a function f is harmonic for such α-stable process (X_t), then the function $Kf(x) = \|x\|^{\alpha-2} f(I(x))$ is again harmonic. We call $f \longrightarrow Kf$ the Kelvin transformation of a harmonic function f.

Let X be a Markov process. We prove that a space inversion property of X implies the existence of a Kelvin transform of X-harmonic or excessive functions and that the inversion property is inherited by Doob h-transform. We also determine new classes of processes having space inversion property amongst transient processes satisfying time inversion property. For these processes, some explicit inversions, which are often not the spherical ones, and excessive functions are given explicitly.

The talk is based on a joint paper with L. Alili, L. Chaumont and P. Graczyk (to appear in Math. Nachr. 2019)